Dual Filtering Data Assimilation of Dynamic Systems

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Introduction

Data Assimilation

- Using field data and modeled data to create a basis for prediction
- To create these "means" we use algorithms: **filters**
  - Ensemble Kalman/Particle
- Ability to take dynamic, multi-variable systems and assimilate
- Commonly found in weather forecasting

[1]

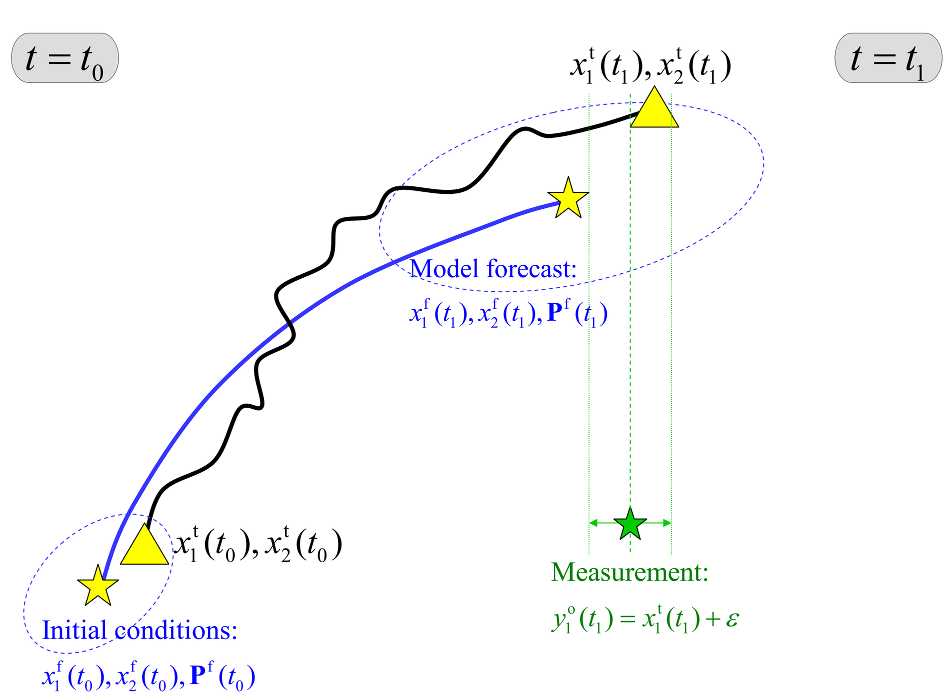


Figure 1: General graphic of data assimilation [1]

Background

Kalman Filter

- Baye's Rule:

$$P(x_t|\psi_t) \propto P(y_t|x_t)P(x_t|\psi_{t-1}) \quad (1)$$

to create Kalman Equation:

$$x^a = x^b - \frac{P^b}{P^b + R}(y - x^b) \quad (2)$$

- Effective for lower dimensional systems
- Higher dimensional systems require an ensemble of x^b's to create an ensemble of x^a's
  - *Ensemble Kalman Filter*

Methods

- Assimilate the Lorenz '96 model:

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b} \sum_{j=J(i-1)+1}^{iJ} y_j \quad (3)$$

$$\frac{dy_j}{dt} = -cby_{j+1}(y_{j+2} - y_{j-1}) - cy_j + \frac{hc}{b} x_{floor[(j-1)/J]+1} \quad (4)$$

[2]

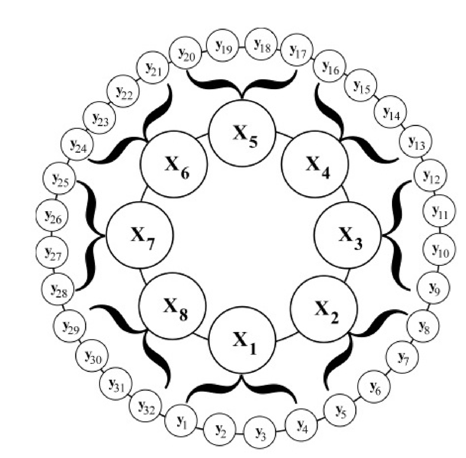


Figure 2: The Cyclical Lorenz '96 System [2]

- Create "field data" y through simulation
- Create an ensemble of x^b's from the Lorenz '96

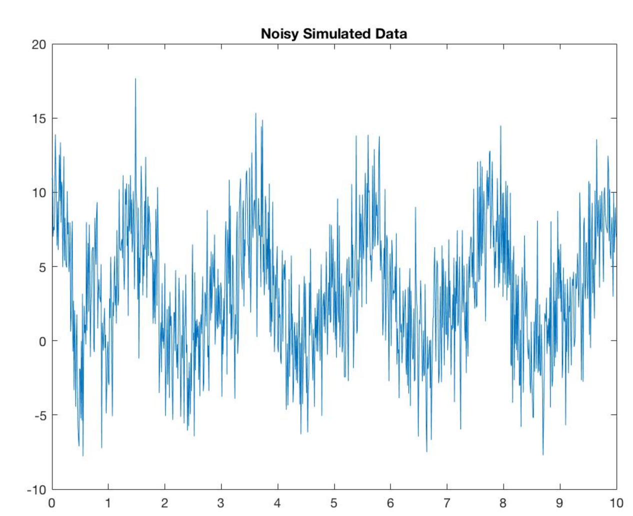


Figure 3: $R = 10$

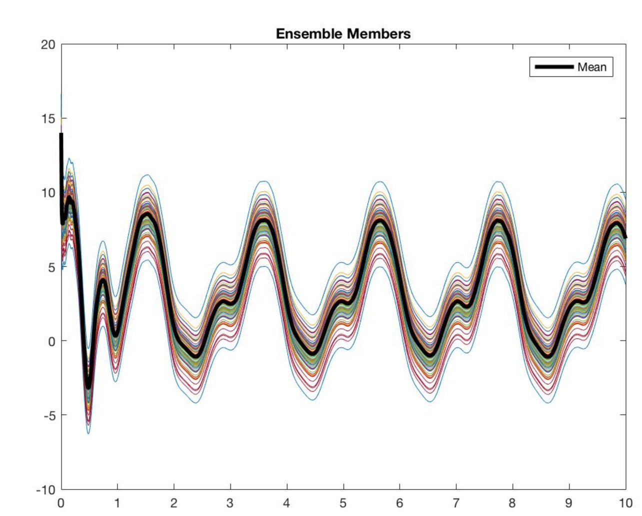
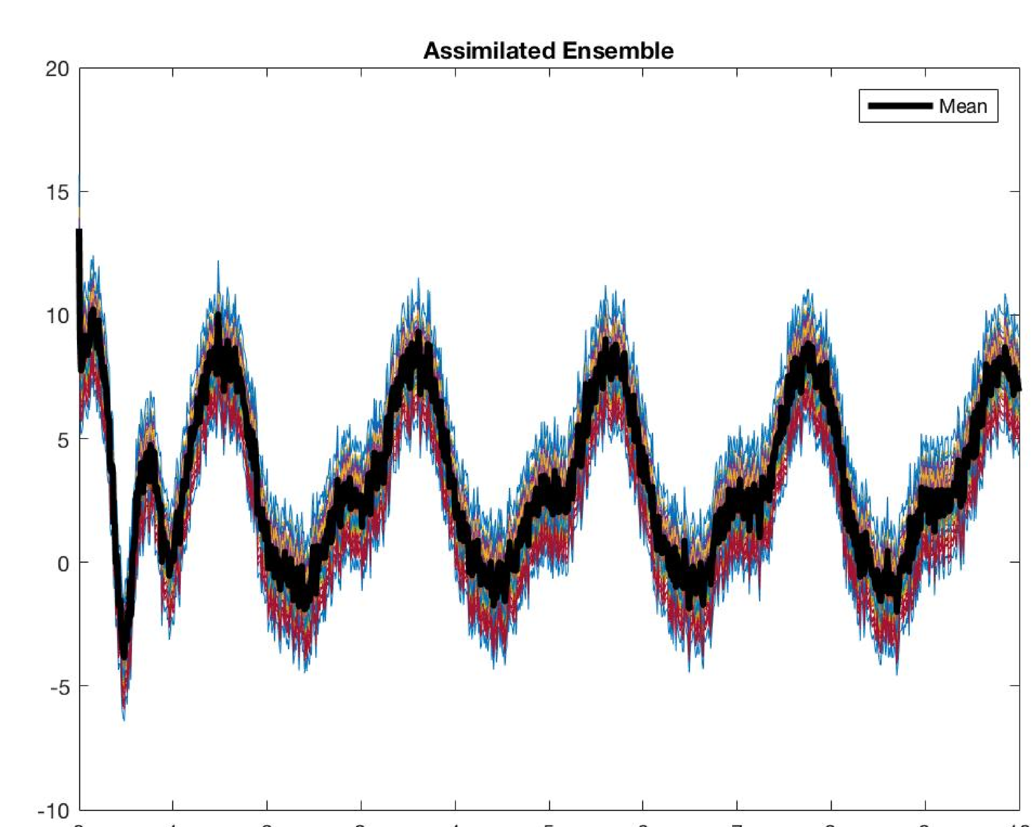


Figure 4: $P^b = 1$

- Assimilate y to the x^b's to create an ensemble of x^a's



Conclusion/Continued Research

- $P^b < R$ then $x^a \approx x^b$
- $R < P^b$ then $x^a \approx y$
- Next Step: Developing a Particle Filter to asses the parameter of Lorenz '96:

$$F = f_0 + \theta_k \sin(\frac{2\pi}{\theta_k + 1} i) \quad (5)$$

[3]

- By filtering, find θ_k's that reduce error in assimilation of Ensemble Kalman Filter

References

[1] Elaine Spiller.
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pages 1–61, May 2017.

[2] Ross M. Leib-Lappen and Christopher M. Danforth.
Aggressive shadowing of a low-dimensional model of atmospheric dynamics.
Physica D, 241(2012):637–648, December 2011.

[3] Naratip Santitissadeekorn and Christopher Jones.
Two stage filtering for joint state-parameter estimation.
Monthly Weather Review, 143:2028–2042, December 2014.

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