Minimizing Total Distance in Sudoku Number Entry

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OBJECTIVE

This research focuses on the effort to reduce the total distance of number entry in a 9 x 9 Sudoku grid. The objective is to model a Sudoku grid and delineate its distances between squares of the same digit containing the numbers one to nine using graph theory in addition to forming a secondary weighted graph.

BACKGROUND

Figure 1: Example of a Latin Square[1]

A	B	C	D
C	D	A	B
D	C	B	A
B	A	D	C

5	11	6	9
6	9	5	11
9	6	11	5
11	5	9	6

- Sudoku originated from a Latin Square, which is a combinational structure.
- There are certain characteristics of Sudoku that have been previously modeled and analyzed using graphs.
- Sudoku has 81 squares where each square is represented by a vertex in the graph.

METHODS

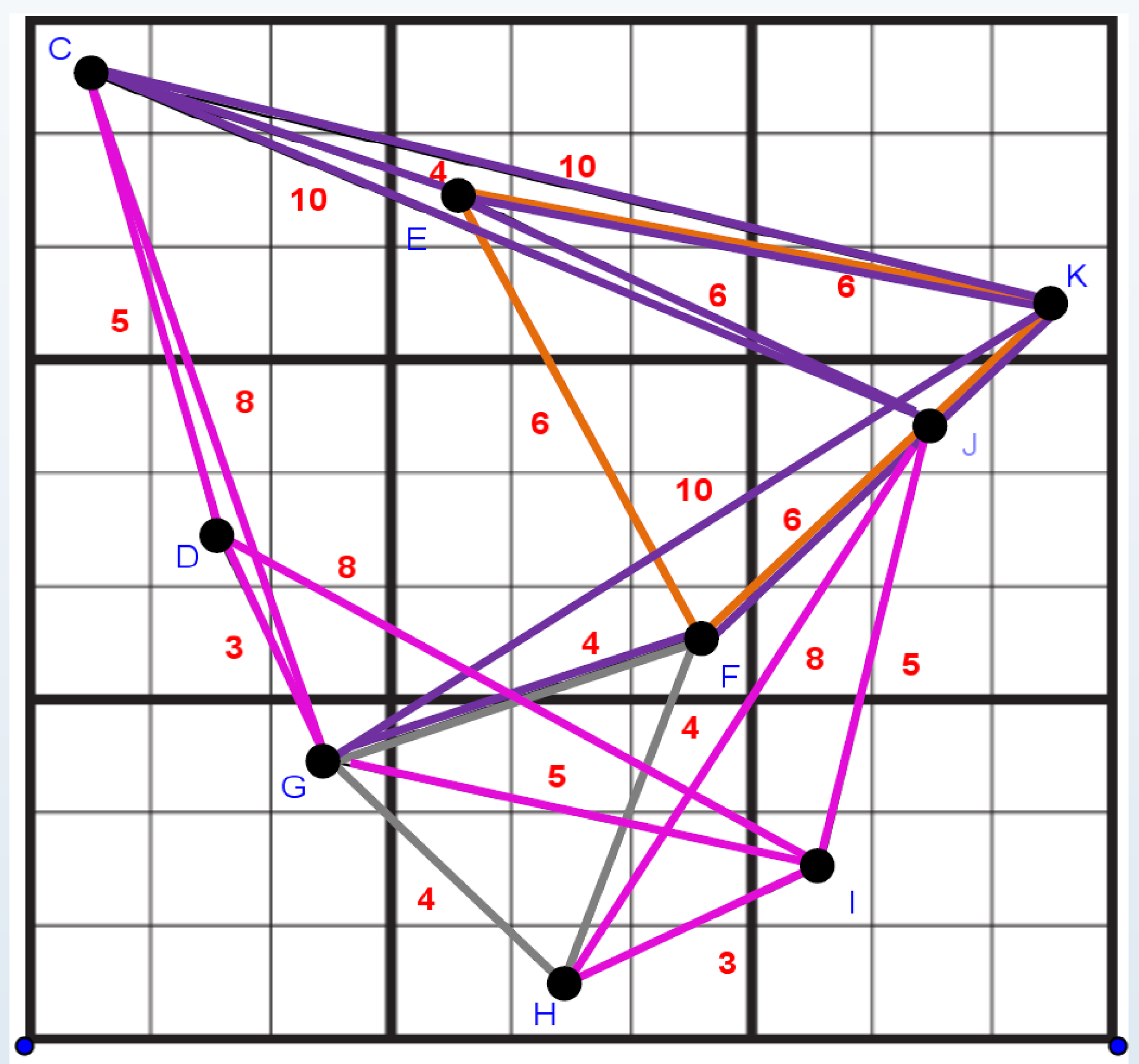
- Graph Theory
  - The use of weighted graphs and paths
- Traveling Salesman Problem

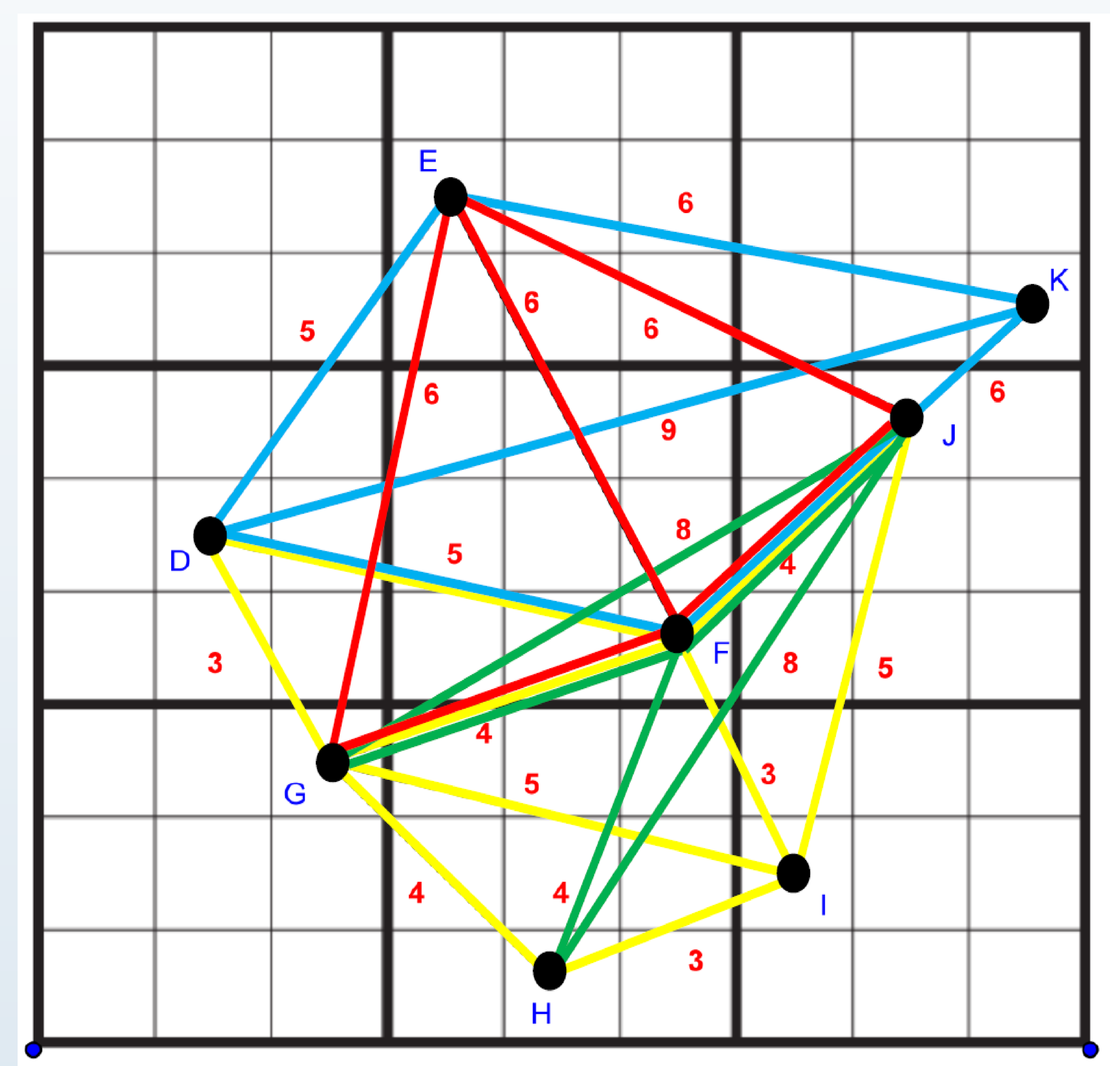
Figure 2: Traveling Salesman Problem Model[2]

RESULTS

Where K=3

K represents the number of blank spaces in the Sudoku grid that we will try to fill in succession. For purposes of this application, they will be assumed to contain the same digit from the numbers 1,…,9.

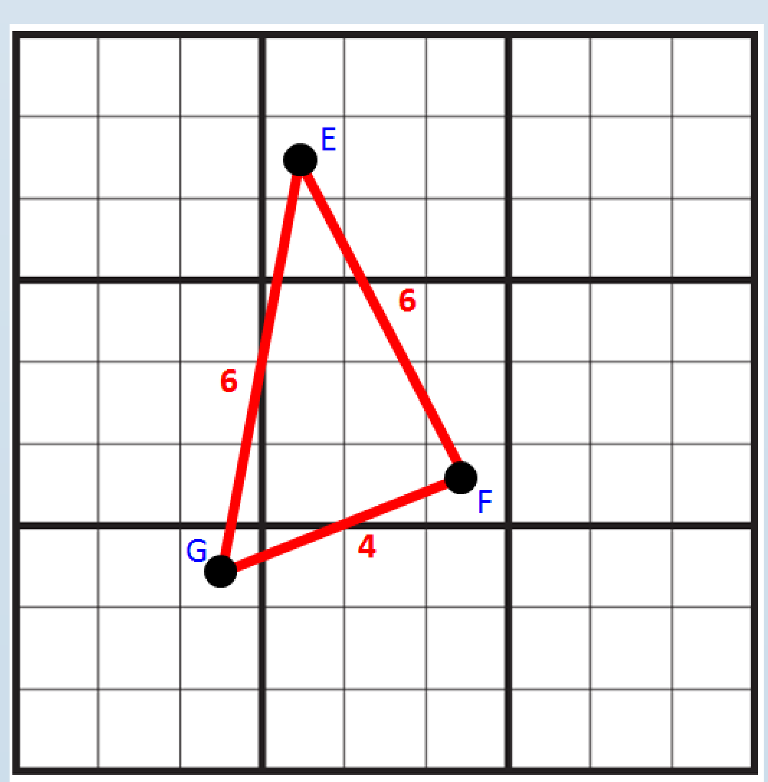




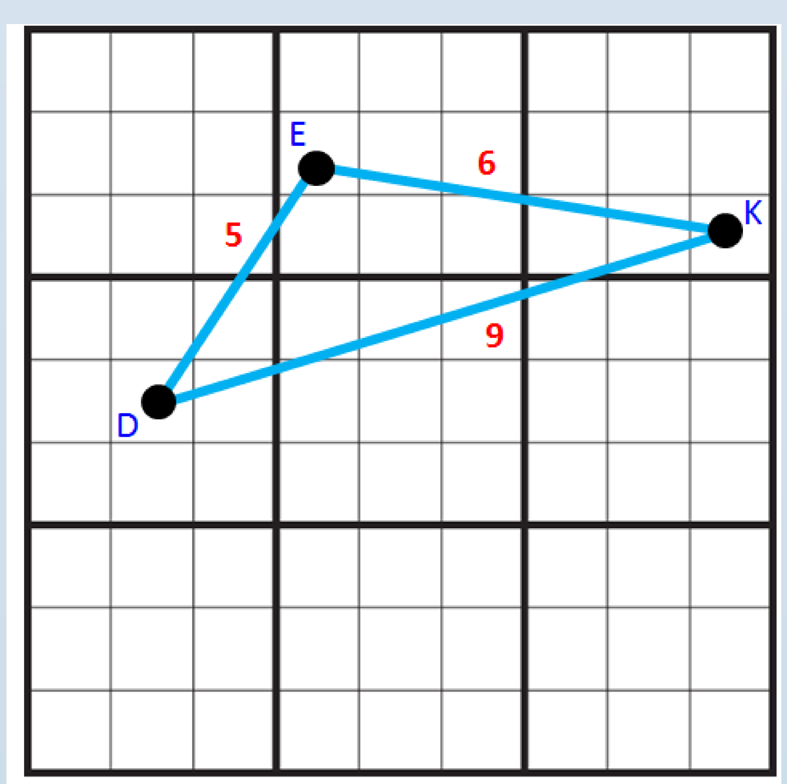
Assumption: Always start at the uppermost corner of the triangle to find its shortest path.

Shortest Paths:

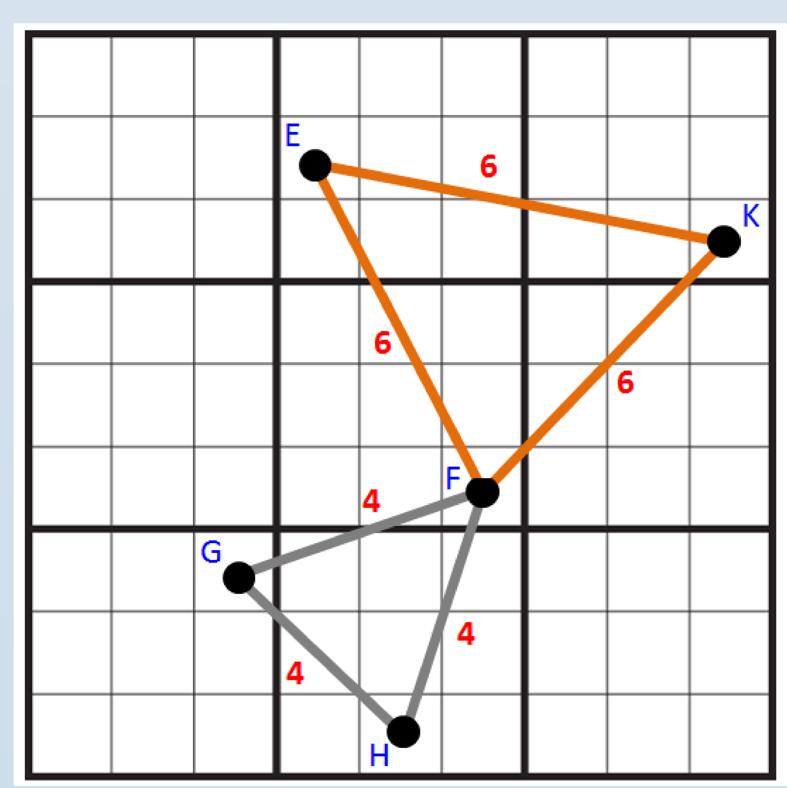
10	ΔCEK, ΔCEJ, ΔKFG
12	ΔEKF
8	ΔFGH
8	ΔCDG, ΔDGI, ΔJIH
7	ΔDGF, ΔFIG. ΔGHI, ΔJFI
10	ΔEGF, ΔEJF
8	ΔJFG, ΔJFH
11	ΔKFD (ΔDEK)



Triangle that allows finding shortest path



Contradiction in finding the shortest path



Equilateral triangles that cannot be expanded

ANALYZING QUESTIONS

1. What can never happen in the configuration?
2. Is there something that will always happen?
3. What is the minimum and maximum distance over all configurations?

CONCLUSION & FUTURE WORKS

It has been concluded that there are potential algorithms and theorems that can be formed with the expansion of configurations. This research is NP-complete in complexity with no known time algorithm to solve. Such findings may also be helpful to the persistent work of the Traveling Salesman Problem.

Future works and goals include:

- Expanding to K=4
- Analyze more configurations to find potential algorithms
- Begin to find the shortest paths using Matlab

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[1] "Latin Square." *Latin Square*. Wikinut Ltd, 2015. Web. 22 July 2015.

[2]"Traveling Salesman Model" *Open Source Physics*. NSF-NSDL, 2014. Web. 19 July 2015.

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