Global Crude Oil Markets and (1,2)-Step Competition Graphs

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Objectives

- Develop a mathematical basis using digraphs, competition graphs & (1,2)-step competition graphs
- Use the mathematical basis to model the global crude oil markets
- Consider what structures in the (1,2)-step competition graph of the model suggest about the markets
- Investigate theoretical questions arising from (1,2)-step competition graphs in different applications; most notably:
  - Determine the minimum number of arcs that can be removed from the digraph that result in the removal of exactly one edge from the (1,2)-step competition graph

Background

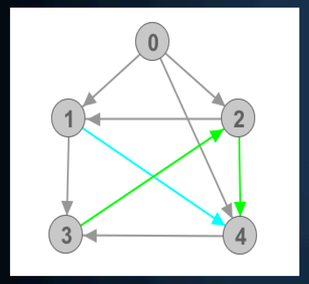
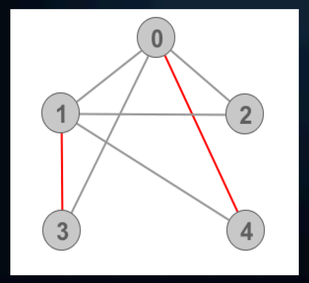


Figure 1. Digraph

A digraph is composed of vertices and directed edges, called arcs

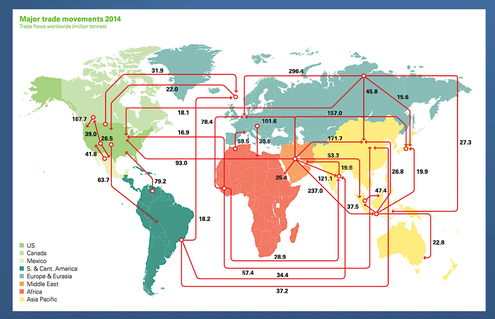


[2] *Figure 2*. (1,2)-step Competition Graph

A (1,2)-step competition graph of a digraph D is composed of:

- *The same vertex set as D*
- *An edge between vertices u & v if u & v are directed toward a common vertex or if u is directed toward a vertex and v can reach the vertex in 2 steps (or vice versa)*

Modeling the Global Crude Oil Market



[1] *Figure 3*. The Global Crude Oil Trading Market in 2014

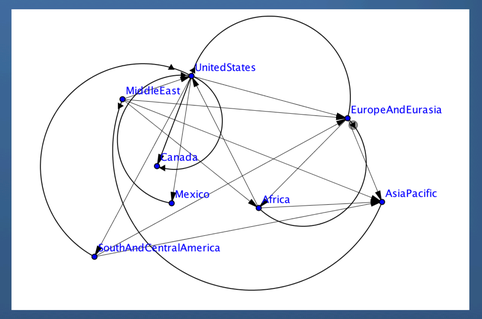


Figure 4. Digraph Representing the Global Crude Oil Trading Market in 2014

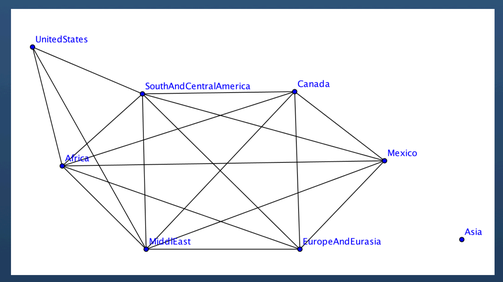
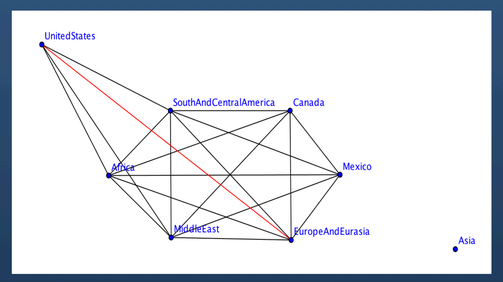


Figure 5. Competition Graph Representing the Global Crude Oil Trading Market in 2014



[2] *Figure 6*. (1,2)-step Competition Graph Representing the Global Crude Oil Trading Market in 2014

Maximizing (1,2)-step Competition Graphs

Determining the number of arcs that are required to be removed from a digraph in order to remove only a single edge from the (1,2)-step competition graph.

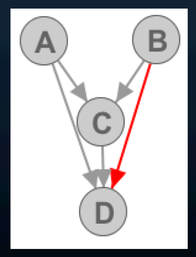


Figure 7. one primary producer; arcs = # of intermediates

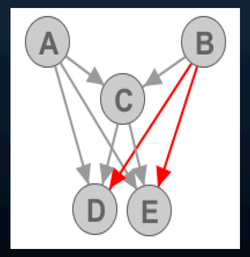


Figure 8. two primary producers; arcs = # of intermediates + 1

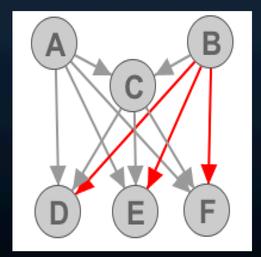
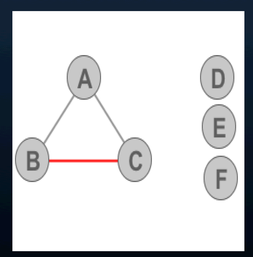


Figure 9. three primary producers; arcs = # of intermediates + 2



[2] *Figure 10*. (1,2)-step Competition Graph

Conclusions and Future Work

Conclusions:

- The global crude oil can be modeled through the use of digraphs, competition graphs, and (1,2)-step competition graphs.
- The number of arcs that need to be removed from the digraph in order to remove one edge from the (1,2)-step competition graph:
  - With one "primary producer," remove the number of intermediates
  - With two "primary producers," remove the number of intermediates plus one arc
  - With three "primary producers," remove the number of intermediates plus two arcs

Future Work:

- Analyze the crude oil trading network accounting for different variables in the network
- Find the minimum number of arcs a digraph must have for the (1,2)-step competition graph to be connected

References

[1] "BP Statistical Review of World Energy – June 2015." *Journal of Policy Analysis and Management* 64 (2015): 19. Web.

[2] Kim A.S. Factor, Sarah K. Merz, The (1,2)–step competition graph of a tournament, Discrete Applied Mathematics, Volume 159, Issues 2–3, 28 January 2011, Pages 100-103

Acknowledgements

All work completed was with the mentorship of Dr. Kim Factor at Marquette University. The National Science Foundation made this relationship possible. Without the support and guidance from Dr. Kim Factor, Marquette University, and the National Science Foundation, the opportunity to gain knowledge in the field of Graph Theory during the summer of 2015 would have never been possible. **NSF Award ACI-1461264.**