Data Assimilation for Fluid Dynamic Models:
Finding Flow Paths of an Object Through Water

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Motivation

Given observational data taken from the trajectory of an instrument flowing in an unknown velocity field, can we recreate the velocity field inducing this motion?

Methods

Data Assimilation is a process that merges observational data with a mathematical model.

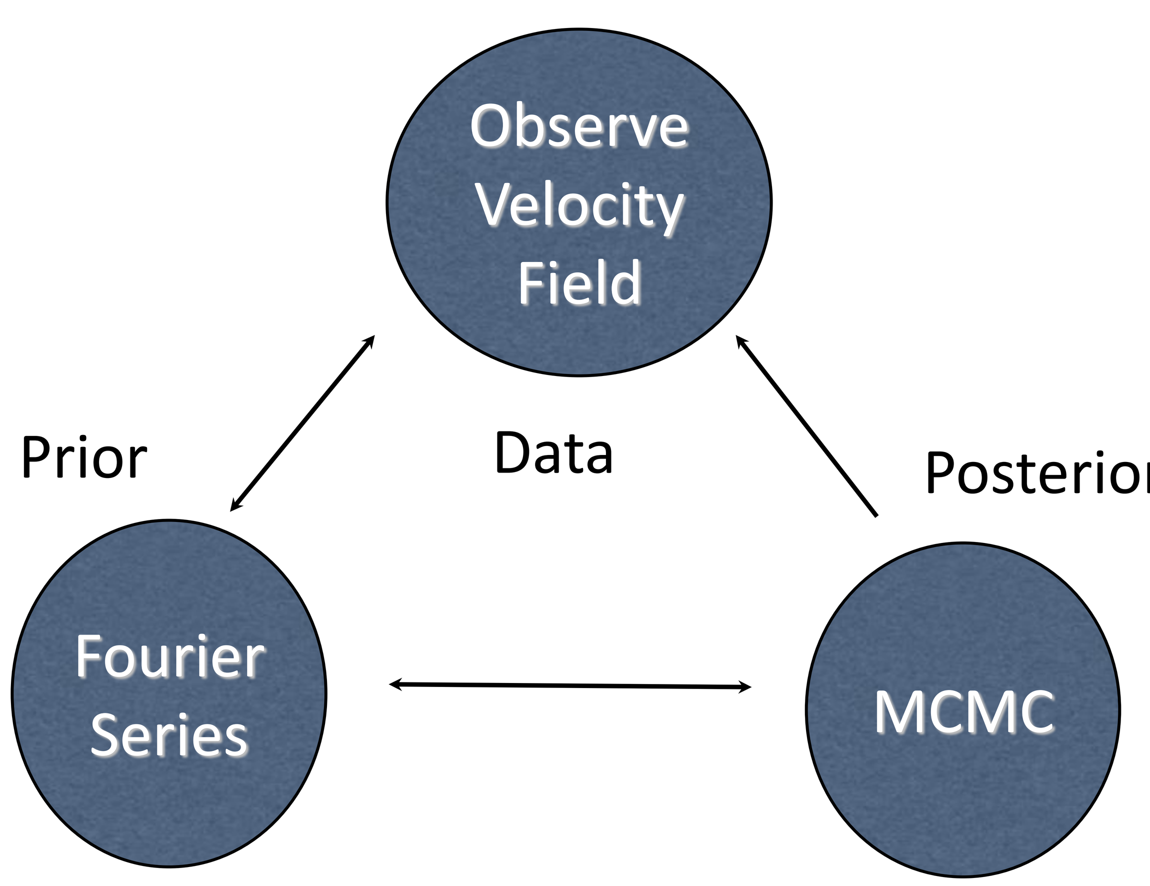
Strategy

Posterior distribution- the probability of unobserved observations conditional on the observed data- on a function space (velocity field) is proportional to the prior distribution multiplied by the likelihood of the data.

Given a velocity field v and data θ**,**

Bayes Theorem: $p(v|\theta) \propto p(v)p(\theta|v)$

p(velocity field | data) ∝ p(Fourier Series)p(data | Fourier Series)
Posterior ∝ Prior x Likelihood



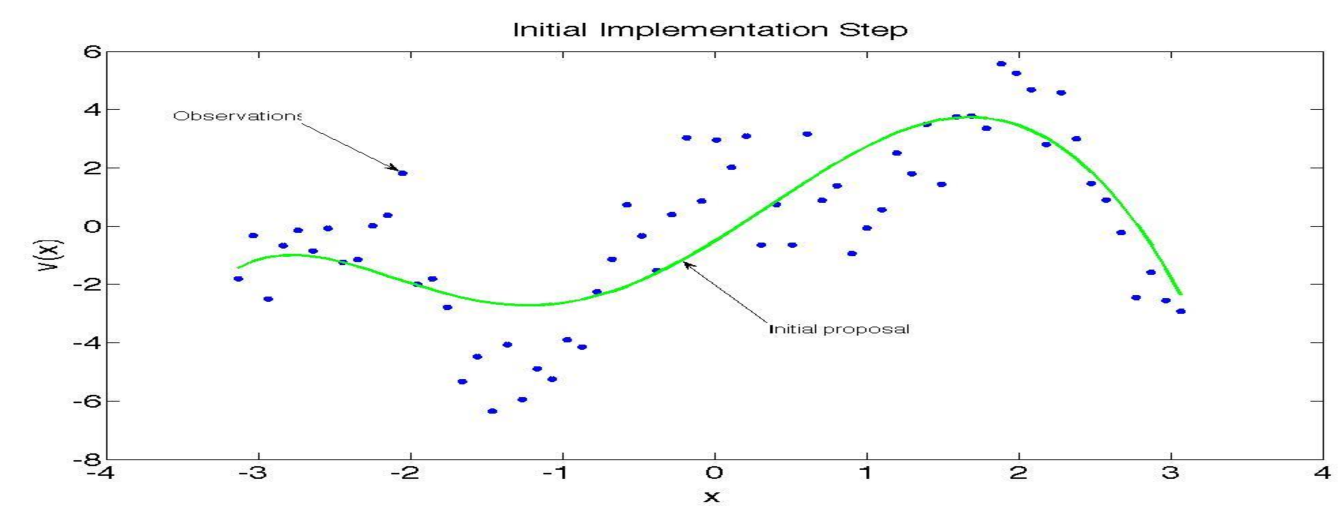
Fourier Series (FS): Most functions can be described as a series of sines and cosines.

$$f(x) = \sum_{n=1}^{\infty} a_n \cos(2\pi xn) + b_n \sin(2\pi xn)$$

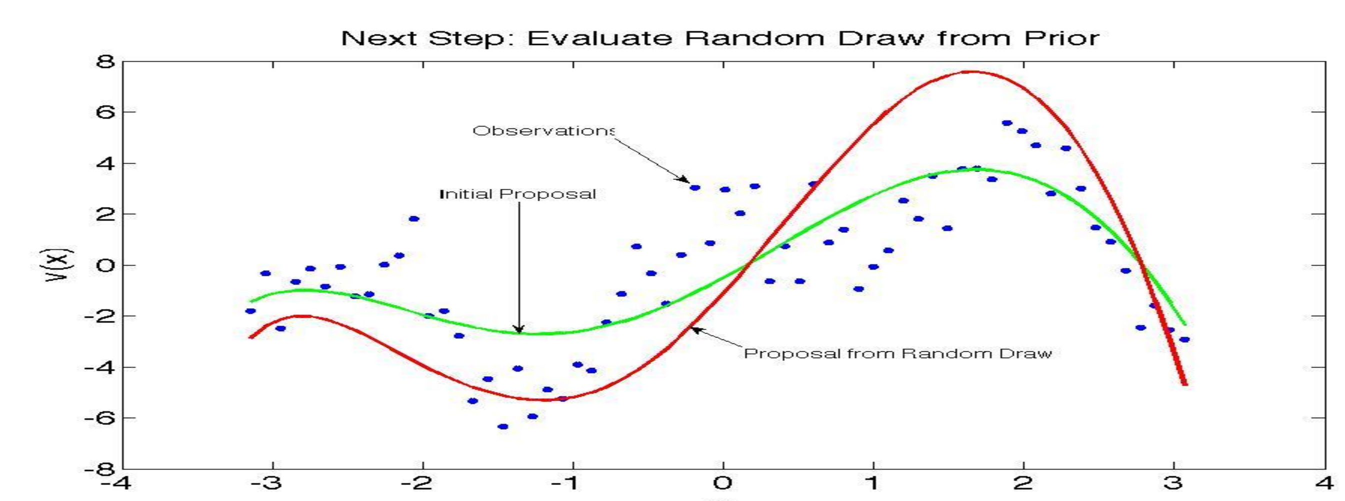
Strategy Continued

1D Sampling Example: Using Eulerian data assimilation where the observations are fixed. Assume a steady flow; velocity does not change with time.

- Low degree polynomial fit of data for 1st proposal



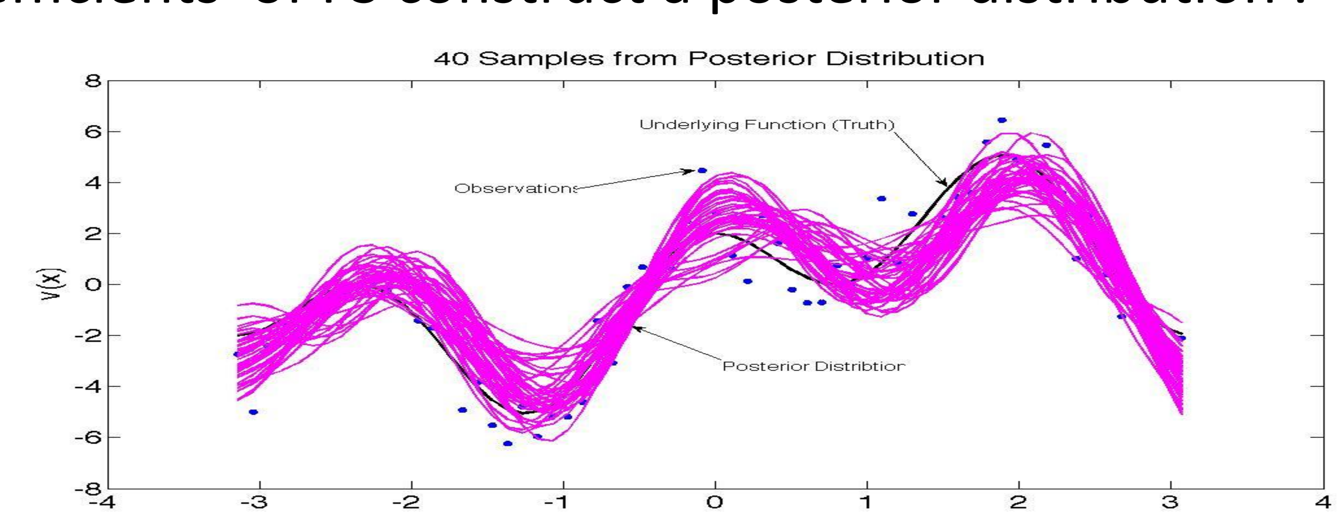
- Prior draws from frequency space, random coefficients (a_n and b_n) of Fourier Series



- Accept proposal v(x) from prior draw if it better fits the observations, else repeat initial proposal. This process is known as Metropolis-Hastings sampling within the Markov Chain Monte Carlo (MCMC) simulation.

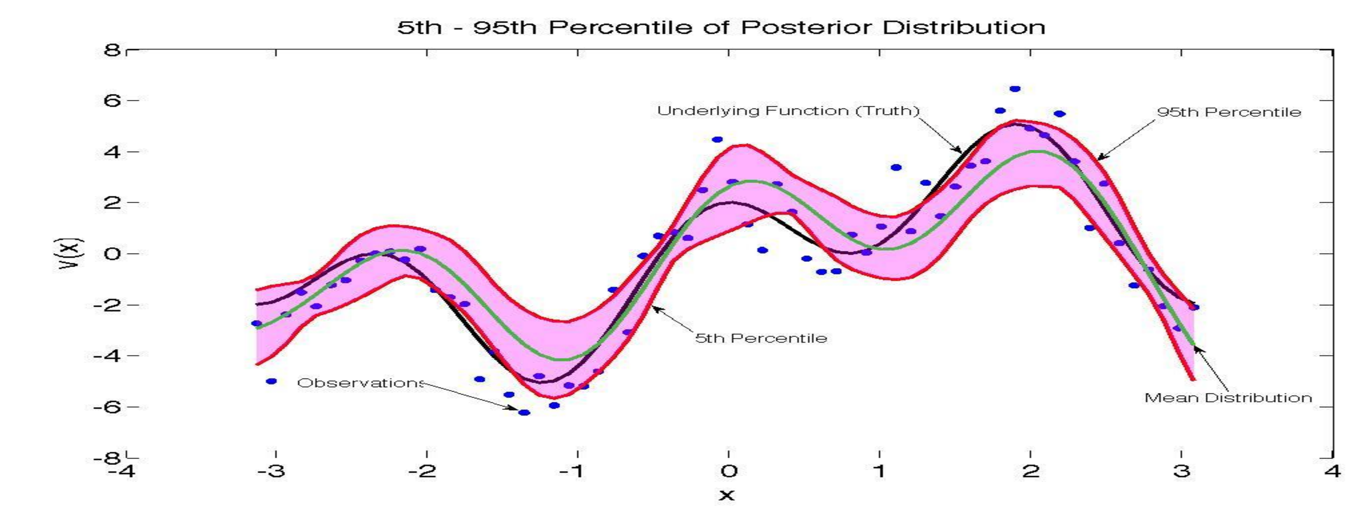
Results: Posterior Sampling

The proposals accepted from the sampling of random coefficients of FS construct a posterior distribution .

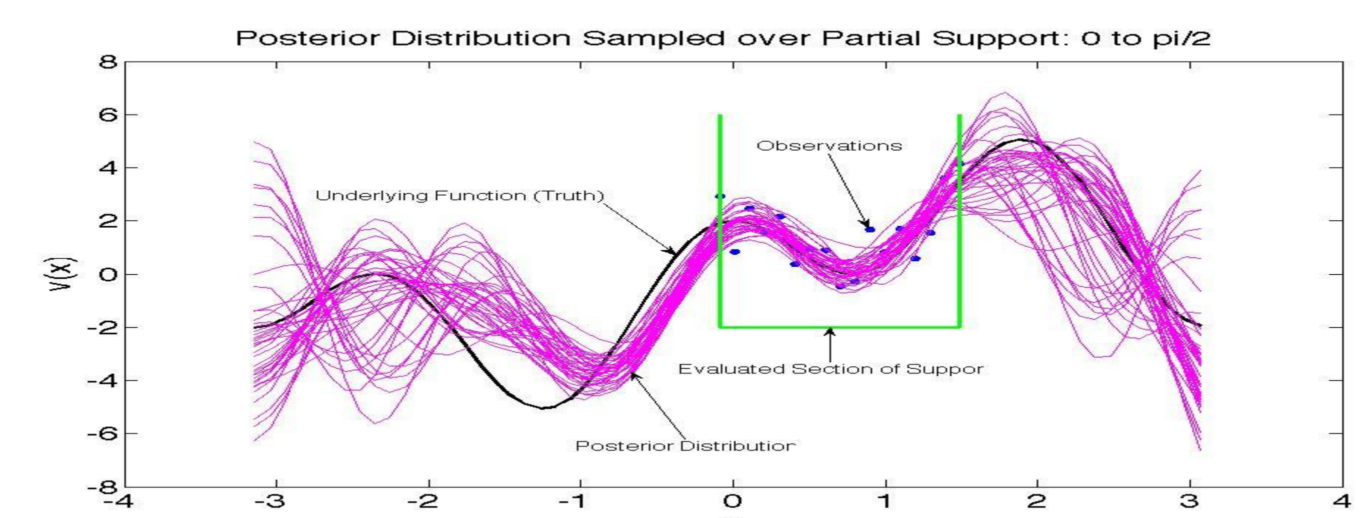


Results: Posterior Sampling Continued

A graph of the area between the 5th and the 95th percentile of the posterior distribution and the mean of the distribution.



A graph showing the posterior distribution conforms to the truth only in the section where observations are available.



Summary & Future Work

The Eulerian 1D sampling performs according to strategy expectations. To achieve the goal of recreating a velocity field from given observational data, there remains the following current & future work:

- Adapt to Lagrangian data-update likelihood
- Adapt the strategy to 2D sampling
- Apply the strategy to data from Dr. Ani Hsieh's lab

Selected References

[1] Damon McDougalll. *Assimilating Eulerian &Lagrangian Data to Quantify Flow Uncertainty in Testbed Oceanography Models*. PhD thesis. University of Warwick. 2012.
[2] Elke Thönnes. Lecture Notes on Monte Carlo Methods University of Warwick.

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