The Nearly Connected (1,2)-Step Competition Graph: A Potential Tool in Food Web Analysis

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Background

❖Food webs map the flow of energy and matter in an ecosystem by describing the feeding relationships between species.

❖Cohen (1968) introduced the competition graph of a food web to model predator-prey relations between species.

❖These graphs have been used as a tool for understanding how an ecosystem may respond to change or what controlled changes can be made in order to obtain desired properties in an ecosystem.

❖Factor and Merz (2010) introduced the (1,2)-step competition graph as an extension of the competition graph.

Objectives

-Extend the theory regarding the competition graph to the (1,2)-step competition graph

-Develop theory regarding the application of the (1,2)-step competition graph as a tool in the analysis of food webs

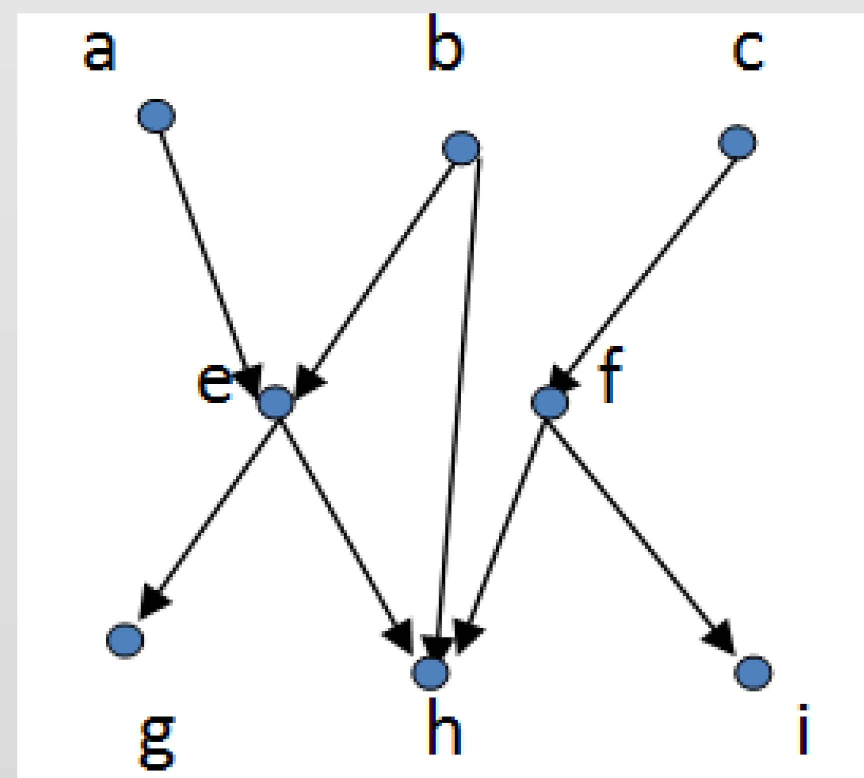
- Characterize digraphs whose (1,2)-step competition graph (generated by all of the vertices in the graph except the basal species) is connected

- Apply the theory to model the competition in the Eco region of the Sahara Desert: The Tibesti-Jebel Uweinat montane xeric woodlands

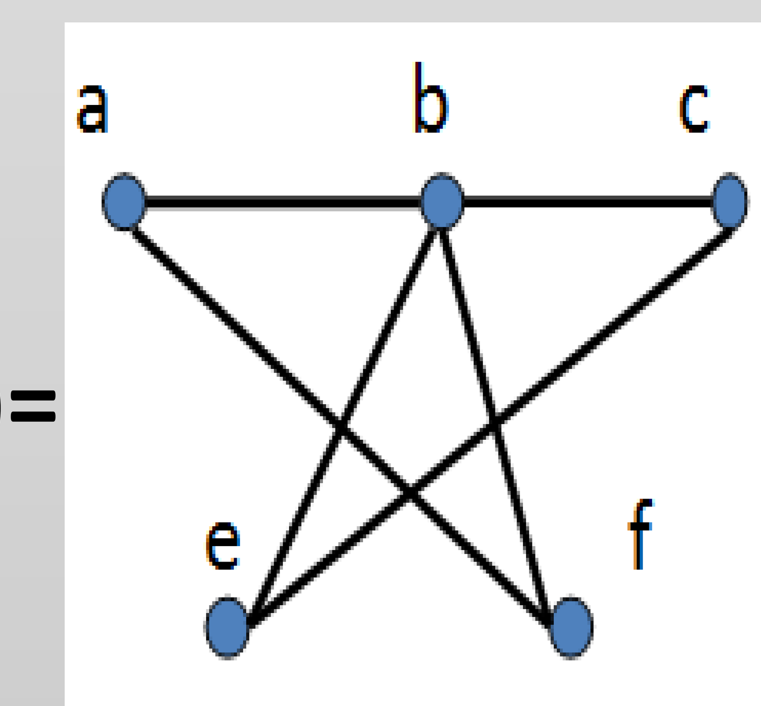
Definitions and Results

Let V= {v: v is a vertex in digraph D}
={a, b, c, d, e, f, g, h, i}
S= {s: s represents a basal species in D}
={g ,h, i}
In this figure D is a subdigraph of the food web compiled for the Tibesti ecoregion

D=



$C'_{(1,2)}(D)$=



$C'_{(1,2)}(D) = C_{(1,2)}(D) - S$

Theorem I

Let d(u,v) be the shortest distance from vertex u to vertex v and $d^-(u)$ be the number of predators preying on species u.

C´ is connected iff for each $v \in V-S$ there exists a distinct $u \in D$ such that dist $(u, v) \leq 2$ and $d^-(u) \geq 2$.

Further Results

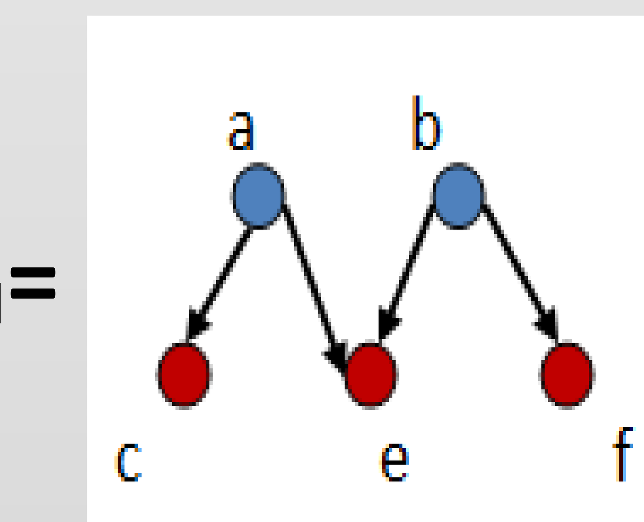
Theorem II:

Let $d^-(v)$ be the number of predators preying on species v and $d^+(v)$ be the number of species on which v preys.

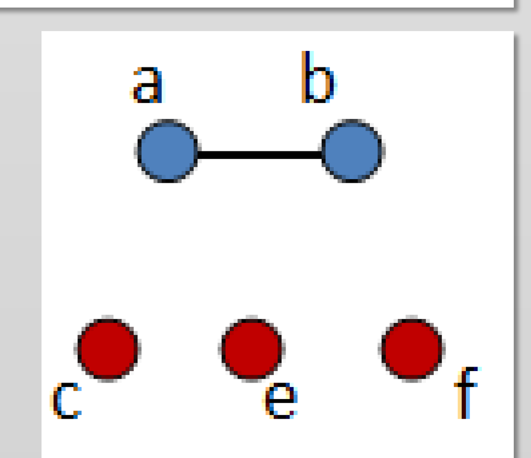
If $d^-(v) = 1$ and $d^+(v) = 0$ then v is isolated in $C_{(1,2)}(D)$.

(Note that v is a primary producer. In digraph D_1 c,e, and f are considered to be primary producers)

D_1=

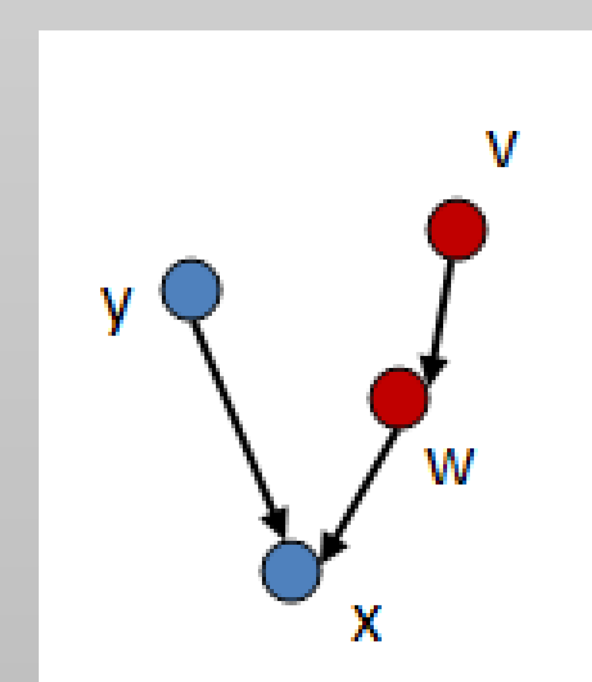


$C_{(1,2)}(D_1)$=

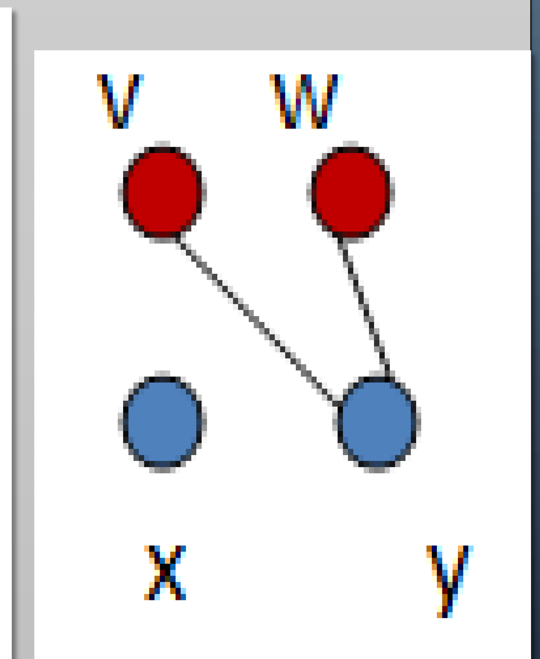


Theorem III:

If $d^+(v) = 1$ and v preys upon w where $d^-(w) = 1$, then (v, w) will not be an edge in $C_{(1,2)}(D_2)$.



D_2



$C_{(1,2)}(D_2)$

Future Work

❖Research what the effects of having predators prey on various trophic levels of specific food webs will have on the completeness of the C´ graph

❖Continue working towards characterizing the digraphs whose C´ graph is complete and/or connected

❖Continue to extend the work that has been done with the competition graph to the (1,2)-step competition graph

❖Apply the theory that has been developed for both types of graphs to food webs to analyze the survivability and/or vulnerability of species in an ecological community

References

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Pimm,Stuart L., Lawton, John H., Cohen, Joel E. "Food Web Patterns and their consequences" *Nature Publishing Group* 25 April.1990.