Finding Relationships Between the Competition and (1,2)-Step Competition Numbers of Acyclic Digraphs

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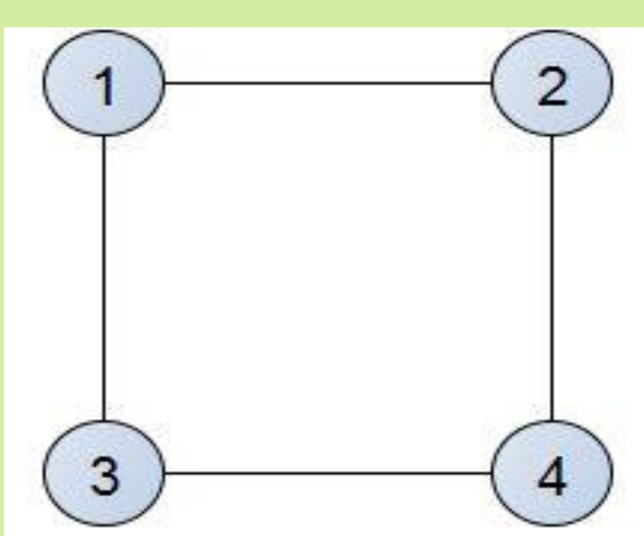
Overview:

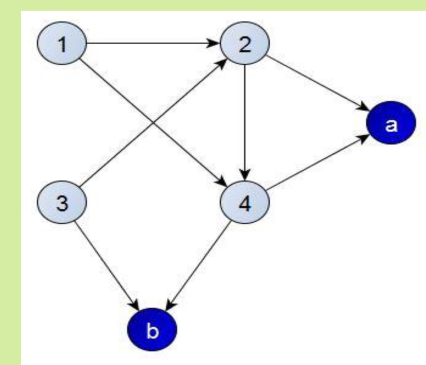
- Current research is based off of previous research done by Factor, Merz, & Sano[2], to answer the question:
  - Are there graphs other than a 4-cycle, C_4, where the competition number of an acyclic digraph D, $\gamma(D)$, is greater than its (1,2)-step competition number, $\gamma_{(1,2)}(D)$?
- Given an acyclic digraph *D*, the **(i,j)-step competition graph of D, $C_{i,j}(D)$,** is the graph with the same vertices as *D* and an edge {u,v} if there exists a third vertex z such that *u* reaches z in at most *i steps and v* reaches z in at most *j* steps
- The **(i,j)-step competition number of G, $\gamma_{(i,j)}(G) = k$**, is the minimum *k* where *G* along with *k* isolated vertices is the (i,j)-competition graph of some digraph
- When $i = j = 1$, we look for the competition number of *G*, and when *i=1* and *j=2*, we look for the (1,2)-step competition number
- The graph C_4 is not the competition graph or the (1,2)-step competition graph of any digraph:



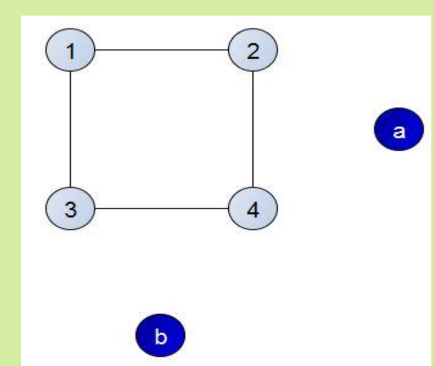
Background:

Previous work found that:

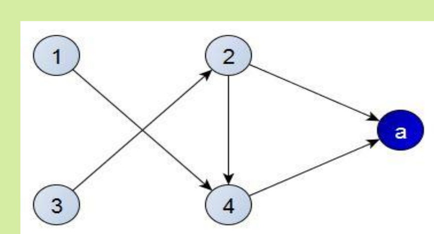
- **$\gamma(C_4)=2$** :



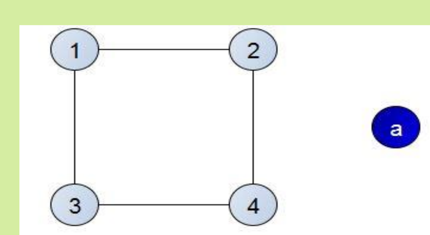
- The competition graph is:



- **$\gamma_{(1,2)}(C_4)=1$** :

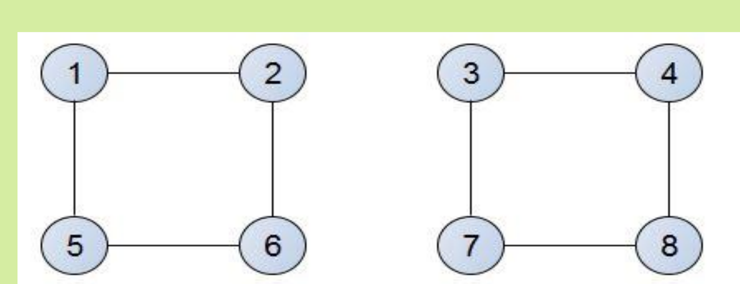


- The (1,2)-step competition graph is:



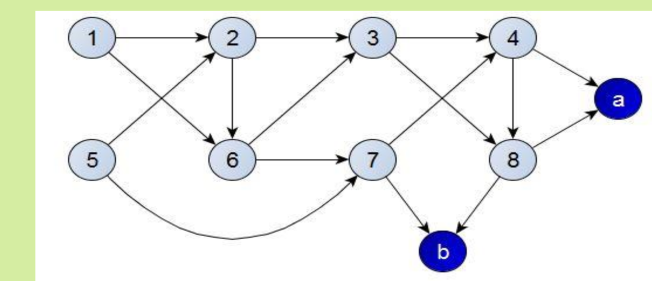
Methods:

- Use the known graph that is not a competition graph as foundation for making new graphs
- Begin by looking at $C_4 \cup C_4$
- Continue on to observe the union of *k* number of C_4 copies

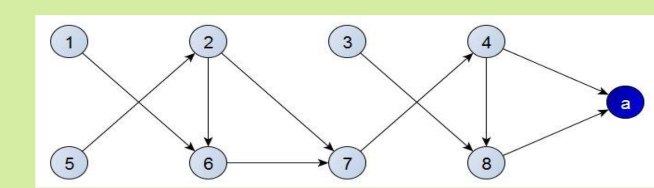


Results:

- **Lemma:** $\gamma(C_4 \cup C_4)=2$ and $\gamma_{(1,2)}(C_4 \cup C_4)=1$
- Digraph with competition graph $(C_4 \cup C_4) \cup K_1 \cup K_1$:



- Digraph with (1,2)-step competition graph $(C_4 \cup C_4) \cup K_1$:



- **Lemma:** For G equal to the union of *k* copies of C_4 , $\gamma(G) = 2$ and $\gamma_{(1,2)}(G) = 1$
- **Theorem:** For the family of graphs where *G* is the union of *k* copies of C_4, $\gamma(G) > \gamma_{(1,2)}(G)$

Future Work:

- Let *G* be the graph that is C_4 with a various number of pendant vertices
- Let *G* be two copies pf C_4 that are connected with one edge, and then two
- See if there is any graph containing C_4 where the two numbers are equal

Acknowledgements:

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Work Cited:

1. Factor, K., Merz, S. *The (1,2)-step competition graph of a tournament*. Discrete Applies Mathematics. Volume 159, Issues 2-3
2. Factor, K., Merz, S., Sano, Y. *The (1,2)-step competition number of a graph*.