Using food webs to study the impact of invasive species

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Introduction

- Hawaii has a very fragile ecosystem due to its geographic isolation and volcanic origins.
- Invasive species are a major threat to native species of the Hawaiian Islands.
- Using food webs allows us to visualize relationships between species.
- Food webs also help us understand the importance of a given species to its ecosystem.
- The study of food webs led me to the discovery of new methods that can be used to examine the impact of invasive species on an ecosystem.

Figure 1: The Ko'olau[1] mountain range is home to many of Hawaii's native species

Background

Definition. *__Food webs__ are a type of digraph where two species are connected in a food web if they have a predator-prey relationship. Let D be a food web, if species x preys upon species y, then $(x, y) \in A(D)$.*

Definition. *Let $C(D)$ be the __competition graph__ of a digraph D and let $x, y \in V(D)$. $xy \in E(C(D))$ if there exists $z \in V(D)$ such that $(x, z), (y, z) \in A(D)$. If $xy \in E(C(D))$, then x and y __compete__ in D.*

Definition. *Let $C_{1,2}(D)$ be the __(1,2)-step competition graph__ of a digraph D and let $x, y \in V(D)$. $xy \in E(C_{1,2}(D))$ if x and y compete in D or there exists $w, z \in V(D)$ such that $(x, z), (y, w), (w, z) \in A(D)$ (fig. 2) or $(y, z), (x, w), (w, z) \in A(D)$ (fig. 3). If $xy \in E(C_{1,2}(D))$, then x and y __(1,2)-compete__ in D.*

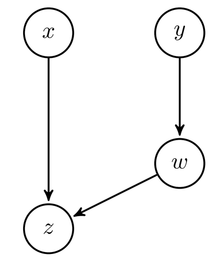

Figure 2

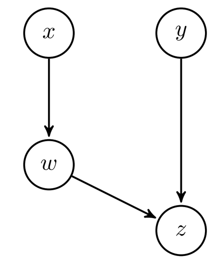

Figure 3

Example

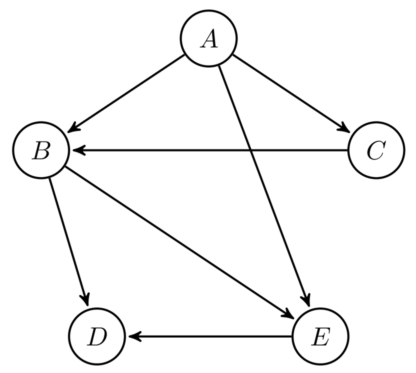

Figure 2: Food Web

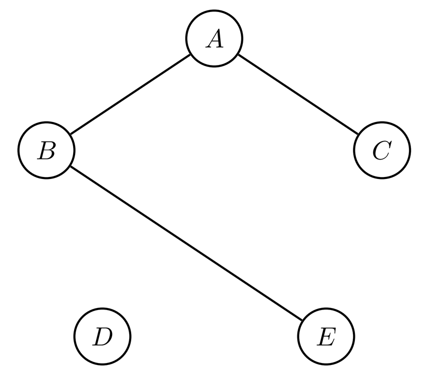

Figure 3: Competition Graph

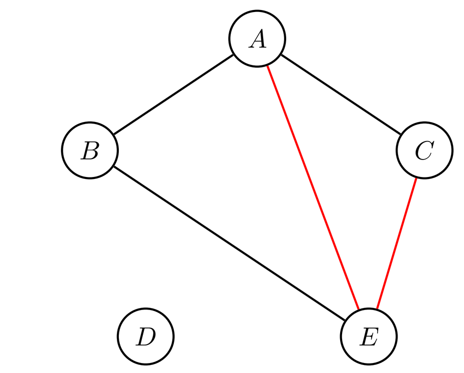

Figure 4: (1,2)-step Competition Graph

Results

Definition. *A __Weighted Food Web__ is a food web with weighted arcs such that the weight of an arc (x, y) is denoted $w(x, y)$ and represents the fraction of y in x's diet. The value of an arc weight must be from 0 up to and including 1 and the weight of all outgoing arcs of a vertex must add up to 1.*

Definition. *A __Weighted Predator Overlap Graph__ is a weighted competition graph G created from a weighted food web D. The __Competition Weight__ is the weight of an edge in a weighted predator overlap graph.*
Let $X = N^+(x) \cap N^+(y)$. Given $xy \in E(G)$, the competition weight is calculated as follows:

$$w(xy) = \sum_{i \in X} \Big(w(x, i) + w(y, i) \Big)$$

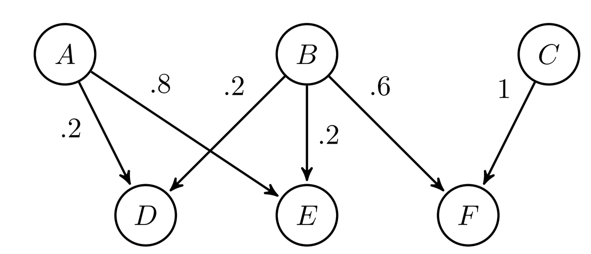

Figure 5: Weighted Food Web

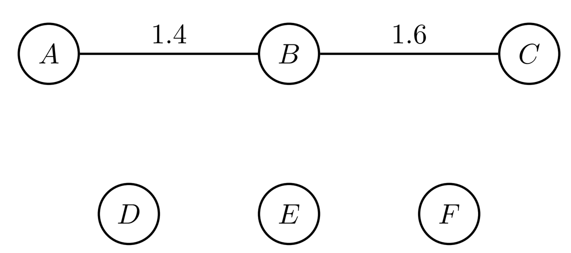

Figure 6: Weighted Predator Overlap Graph

Theorem. *The competition weight of any edge in a weighted predator overlap graph is at most 2.*

Definition. *The __Benefit Digraph__ of D is denoted $B(D)$ and is created by adding an arc (x, y) to the arc set of $B(D)$ if there exists $w, z \in V(D)$ such that $(x, w), (w, z), (y, z) \in A(D)$.*

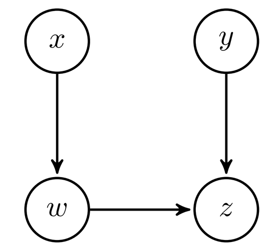

Figure 7: Food Web

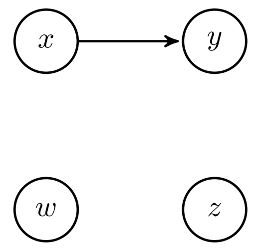

Figure 8: Benefit Digraph

Theorem. *There exists an arc between two vertices in a benefit digraph if and only if the vertices (1,2)-compete in a weighted food web.*

Definition. *The __Vertex Weight__ of a vertex x in a food web is denoted $w(x)$ and represents the growth rate of a species. If $w(x) = 0$, then the population of x is constant. If $w(x)$ is positive, then x is increasing in population and if $w(x)$ is negative, then x is decreasing in population.*

$w(A) = 0$
$w(B) = 0$
$w(C) = 0$
$w(D) = 0$

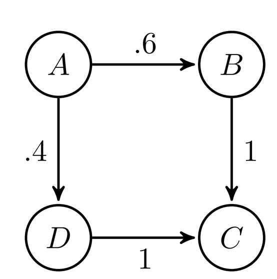

Figure 9: Homeostatic Food Web

$w(A) = 0$
$w(B) = 0.3$
$w(C) = -0.3$
$w(D) = 0$

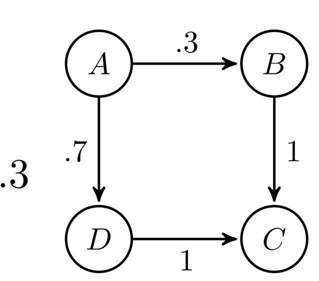

Figure 10: Food Web with weighted vertices

Definition. *A __Homeostatic Food Web__ is a food web such that the vertex weight of each vertex in the food web is 0.*

Conclusions

- Food webs of ecosystems are often very complex.
- Having a variety of tools assists in interpreting these food webs.
- Weighted graphs give us a way to empirically analyze relationships between species.
- These methods can be used for any ecosystem and are especially useful when studying invasive species.

Figure 11: The native 'I'iwi[7] is in danger of extinction

Future Work

- Define weighted benefit digraph.
- Find method to calculate vertex weight after change in food web
- Test accuracy of new methods by applying them to past ecosystems.
- Explore applications of benefit digraphs in areas outside of ecology.

Figure 11: The Haleakala Silversword[8] has benefited from recent conservation efforts

References & Acknowledgements

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