Minimum i, k values to Generate a Compete (i,k)-Step Competition Graph

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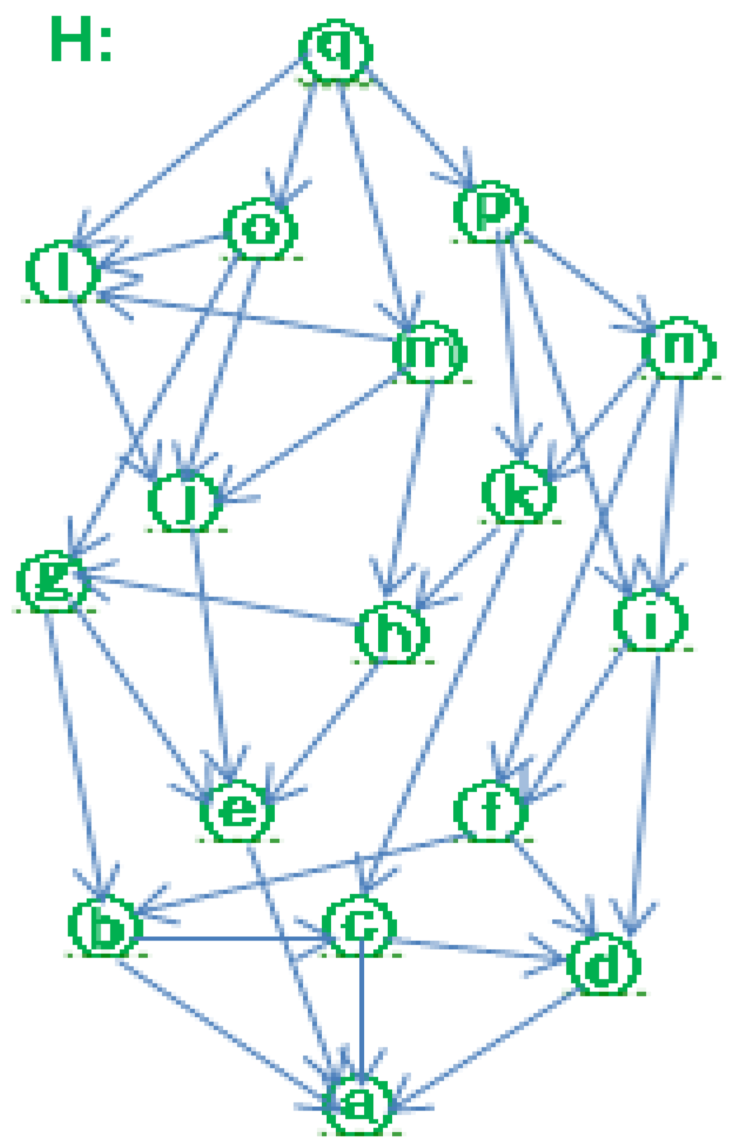
Introduction:

- Given the food web of a particular ecosystem, Ecologist can depict the connections between different predators and prey through the use of competition graphs. Competition graphs can show the connectedness or disconnectedness of species with in an ecosystem. Knowing the connections between species allows ecologists to analyze and make predicts about an ecosystem.
- In a competition graph, species can be connected by direct competitions as well as indirect competitions.
- The focus of this project is as follows: When given an acyclic digraph in which the underlying graph is regular, how can one determine the minimum i and k values such that the (i,k)-step competition graph of the digraph represents all possible direct and indirect competitions?

Definitions:

- *Competition Graph.* Given a digraph, D, the competition graph of D, C(D), is a graph on V(D) such that $\{x, y\}$ is an edge in C(D) if and only if the outset of x and the outset of y share a common element. [1]
- *(i,k)-Step Competition Graph.* Given a digraph D, the (i,k)-step competition graph of D, $C_{i,k}(D)$, is graphed on V(D) such that $\{x, y\}$ is an edge in $C_{i,k}(D)$ if and only if z is a vertex in D not equal to x or y such that $d_{D-y}(x,z) \leq i$ *and* $d_{D-x}(y,z) \leq k$ *or* $d_{D-y}(x,z) \leq k$ *and* $d_{D-x}(y,z) \leq i$, and $i \leq k$. [1]
- *Longest Shortest Path.* After recording the shortest path from each vertex of a digraph to a basil species, I refer to the path with the largest length as the longest shortest path.
- *Basil Species.* A basil species, sometimes referred to as a primary producer, produces it's own energy and, therefore, does not prey on any species. In a digraph, a basil species is represented by a vertex whose out-degree is zero.
- *Top Predator.* A top predator is an animal in an ecosystem that has no predators. In a digraph, a top predator is represented by a vertex whose in-degree is zero.

H:



Shortest Path to a:

b	1	j	2
c	1	k	2
d	1	l	3
e	1	m	3
f	2	n	3
g	2	o	3
h	2	p	3
i	2	q	4

Average Shortest Path	2.18
i-value	3

Longest Path to a:

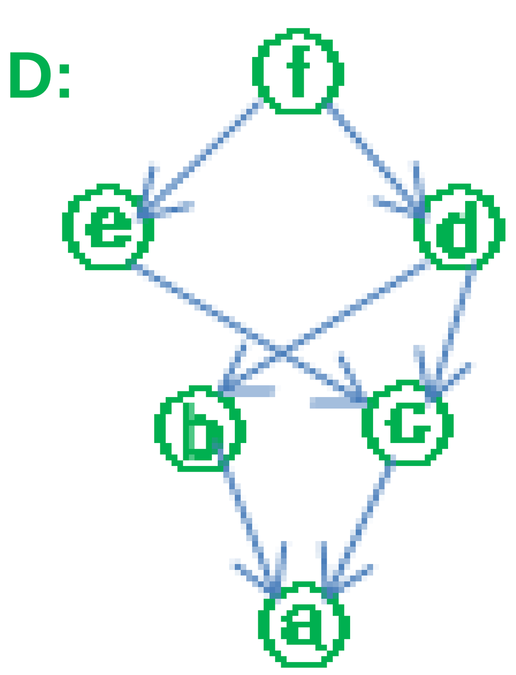
b	3	j	2
c	2	k	3
d	1	l	3
e	1	m	4
f	4	n	6
g	4	o	5
h	5	p	7
i	5	q	8

Average Longest Path	3.93
k-value	4

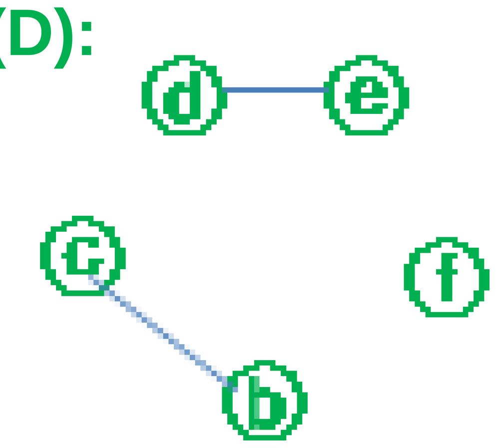
$C_{3,4}(H) = K_{16} - [\{b.c\} + \{h,l\} + \{l,k\}]$

While $C_{3,4}(H)$ contains all possible edges, $C_{2,4}(H)$ also contains all possible edges.

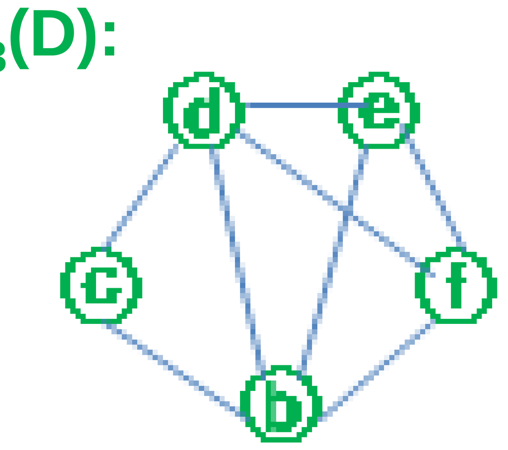
D:



C(D):



$C_{1,3}(D)$:



References:

- [1] K. A. S. Factor and S. K. Merz. The (1,2)-step competition graph. Discrete Applied Mathematics, 2011.

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Challenges:

- Certain (i,k)-step competition graphs can never be complete.
  - Vertices who do not have paths to the same basil species will never compete.
  - If a vertex that does not beat a basil species and has an out-degree of just one, then the vertex will never compete with the element of its outset.
- Multiple variables to consider:
  - Regular number of underlying graph
  - Number of Vertices
  - Number of Top Predators
  - Number of Basil Species
  - Number of species that will never compete

Results:

- Given an acyclic digraph, D, in which the underlying graph of D is n-regular, the number of top predators in D is one, and the number of basil species in D is 1, $C_{i,k}(D)$ contains every possible edge if:
  - i is the least integer greater than the average of the lengths of the shortest path from each vertex to the basil species,
  - k is the least integer greater than the average of the lengths of the longest path from each vertex to the basil species,
  - k is greater than or equal to the length of the longest shortest path,
  - Each vertex with more than one path to the basil species has at least two paths to the basil species of length less than or equal to k.
- Given each parameter is true, $C_{i,k}(D)$ will represent every possible direct and indirect competition; however, i and k may not necessarily be the smallest possible values needed to represent each possible competition in D.
- These parameters accurately depict my research; however, there is no official proof.
- While checking each of these parameters may take longer than desired to reach a result, this method of checking i and k values needed for the most complete (i.k)-step competition graph is much more efficient than creating each step competition graph by hand.