Data Assimilation for Fluid Dynamic Models

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Abstract

We are proposing Sequential Monte Carlo methods as Lagrangian data assimilation technique for ocean gliders. Our goals is to estimate velocity field and vortices path based on simulation results and observational data. We developed a particle filter to perform assimilation on point-vortex systems. We chose to test these methods on two-layers point-vortex system because of their highly non-linear dynamics that presents challenges to many other data assimilation techniques.

Point-vortex system

- **Vorticity:** tendency for elements of a fluid to spin
- **Vortex:** discrete point in a layer of fluid inducing velocity field around its axis as it moves along its path.
- **Point vortex system:** system with zero vorticity everywhere except at a finite number of points (vortex).
- **Characteristics:**
  - **λ:** radius of deformation of a layer.
  - **Γ:** vortex strength of circulation.

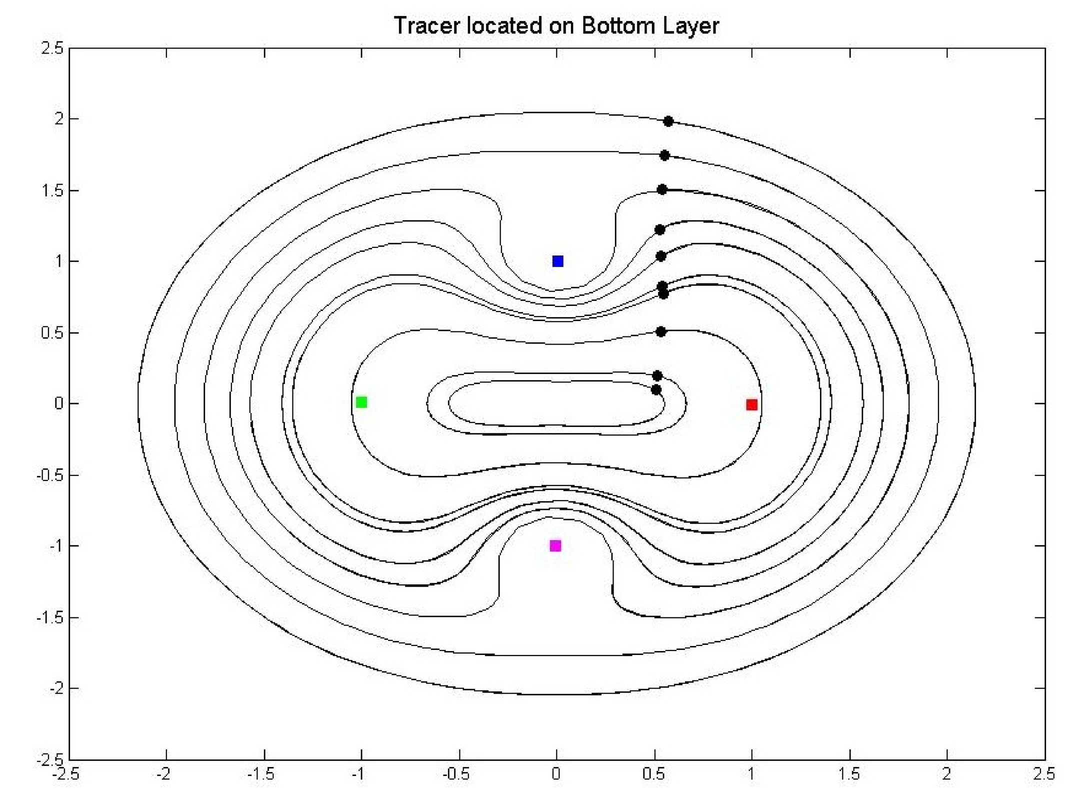
Particle filter

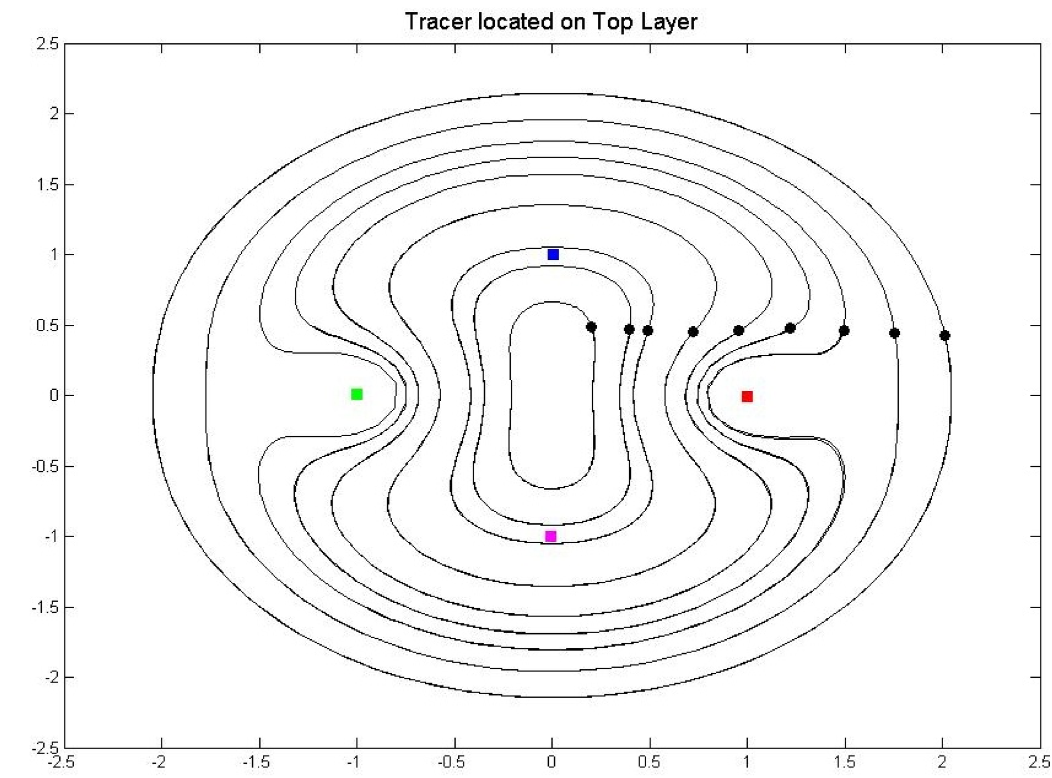
Sequential Monte Carlo (SMC) filtering:
data assimilation technique that estimates a vortex and drifter location based on a weighted average of predictions of an ensemble of sample particles.

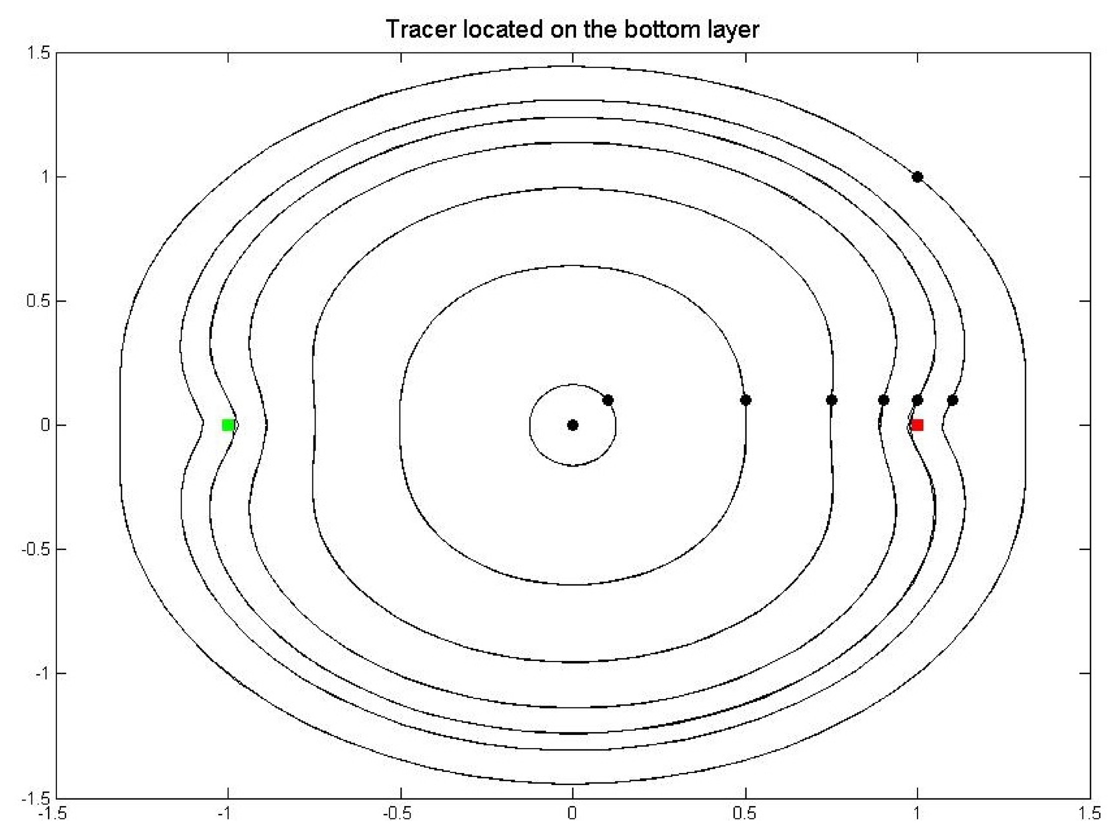
How it works:
a) Start with a cloud of *N* sample particles within an observational uncertainty of known initial conditions.
b) Run the particles forward in time until an observational time.
c) Make an observation of the drifter's location (with known uncertainty) at the observational time
d) Attribute a weight to each sample particles depending on how close its drifter is to the observation.
e) Repeat the process from (b) to (d).

➔ The predicted location of a vortex and/or drifter at time *t* is the weighted average of the locations of the vortices and/or drifters of the sample particles at time *t*.
➔ The particle filter can fail if only a few samples particles are close to an observation.

Velocity fields in 2 layers point-vortex systems







Experimental setup

- **Efficiency test**
  - To test the efficiency of our particle filter, we performed a series of 500 test runs assimilating the same data and observing the filtering error fluctuation.
  - These tests were performed on 2 layer point-vortex systems with 2 and 4 vortices.
- **Filtering parameters**
  - Time step = 1/200 secs
  - Number of particles = 400
  - Filtering duration = 60 secs
  - Resampling factor = 10 %
  - Observation noise = 0.02
  - System noise = 0.02

Implementation

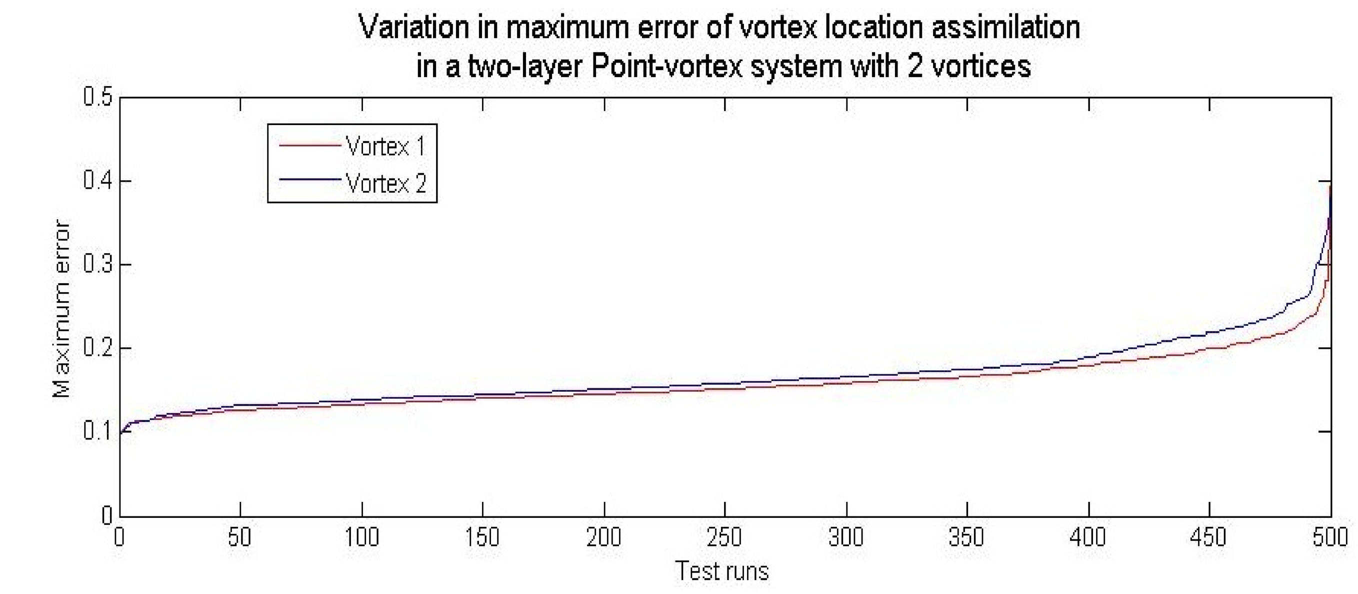
The following algorithm was ran 500 times in an attempt to analyze how many times the particle filter fails over several runs.

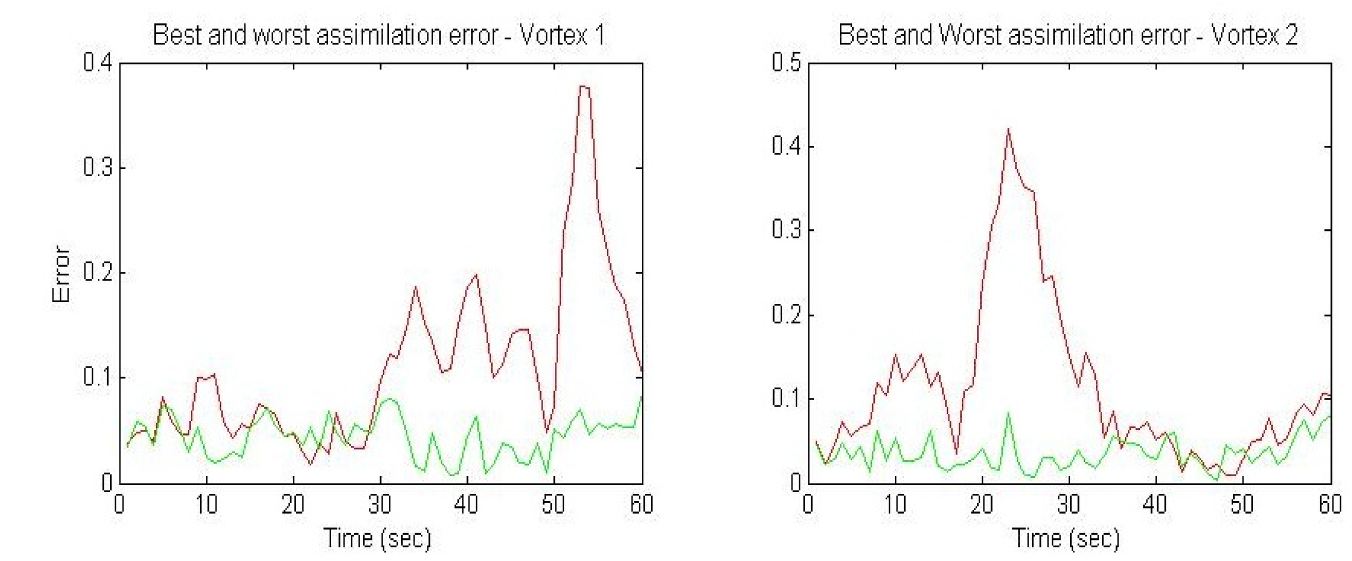
Setup
1. Generate a "truth" run of the system with given initial conditions.
2. At each observation time, record the position of the drifter with an observation uncertainty θ_{obs}.

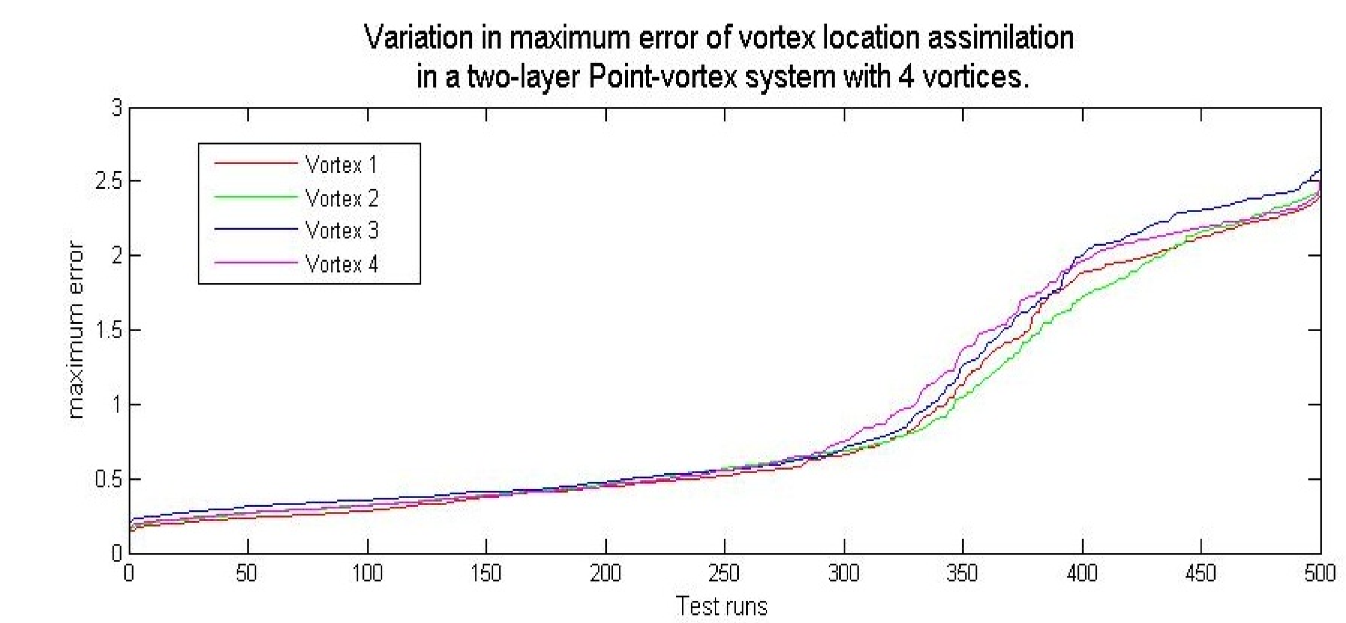
General Filtering Algorithm
3. Generate a cloud of *N* particles within θ_{obs} of the initial conditions.
4. Run forward the *N* particles according to the dynamics of the system.
5. If an observational time is reached, attribute a weight to each particle depending on how close it is to the observation. Closer particles receiving higher weights.
6. If at anytime the weight distribution degrades, consider resampling the ensemble of of particles.
7. Repeat step (4) through (6).

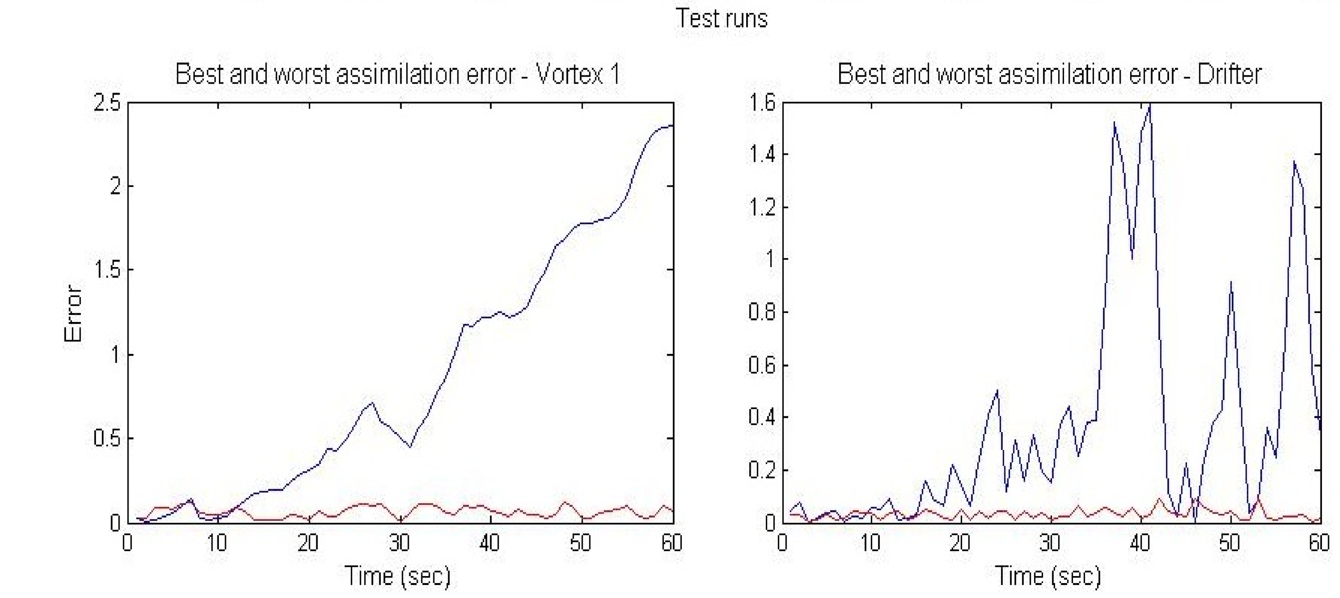
Resampling algorithm
8. Consider a percentage (Re-sampling factor) of the population *N* with large weights.
9. Duplicate each particles in the above selected population proportionally to its weights until a new population *N* is formed

Filtering results









References

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2.E.T. Spiller *et al.*
Modified particle filter methods for assimilating lagrangian data into a point-vortex model. Physica D: Nonlinear Phenomena, 237(10-12):1498 – 1506, 2008.