Predicting Flows with Filters and Ergodicity

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Introduction

Our goal is to create a means to more accurately predict flow trajectories based on partial state observation.

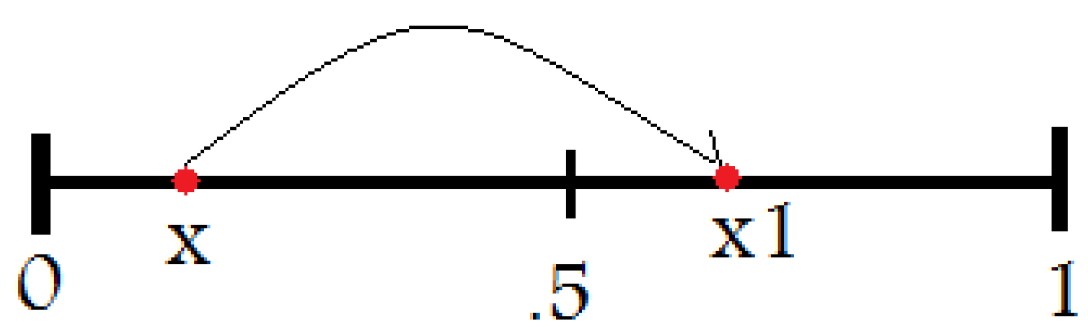
In order to accomplish this task, we are looking into modifying a particle filter to make it more robust by adding criteria to the filter, and checking the consistency with ergodic theory.

Jack has been working with Dr. Scott on ergodic theory
Gabe has been working with Dr. Spiller on particle filters

Erodicity

What is Ergodicity?
Samples the entire space

·Ergodic: if a map is said to have ergodicity, we say that it is ergodic



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If ergodic, map would hit many points over time

To determine ergodicity, split space according to scale s
If s = 1, space in 2 parts (pictured above)
s = 2, space in 4 parts
s = 3, space in 8 parts
s = s, space in 2 to the power of s parts

Then, as point moves through time, count the number of times the point "lands" in each subinterval

This information is used to compute the time average, an average amount of times the point lands in each subinterval. The time average is then compared to the space average, an average of the length of each subsection computed as 1/(2^s). This comparison yields the **ergodic defect**.

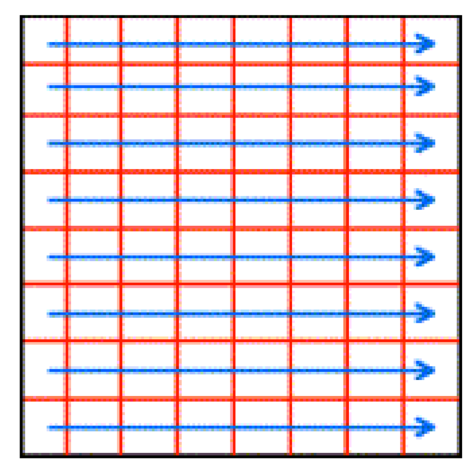
Traditionally in ergodic theory, a map either is or is not ergodic. We use an ergodic defect to quantify *how* ergodic a map is.

-Ergodic Defect: captures deviation from ergodicity - i.e., indicates the extent to which the map/flow deviates from being ergodic

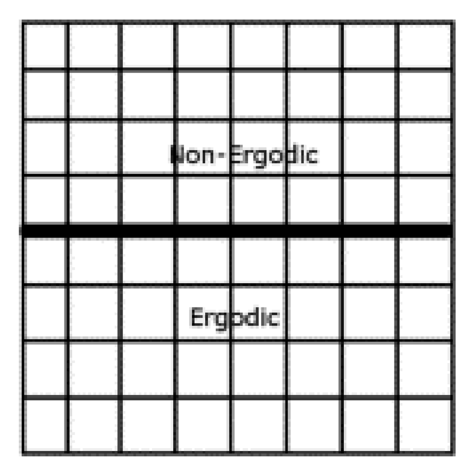
Wrote code in MATLAB to test ergodicity and compute ergodic defect

Tested two 2-dimensional maps:

Invariant Circles
y doesn't change
x moves across space from left to right

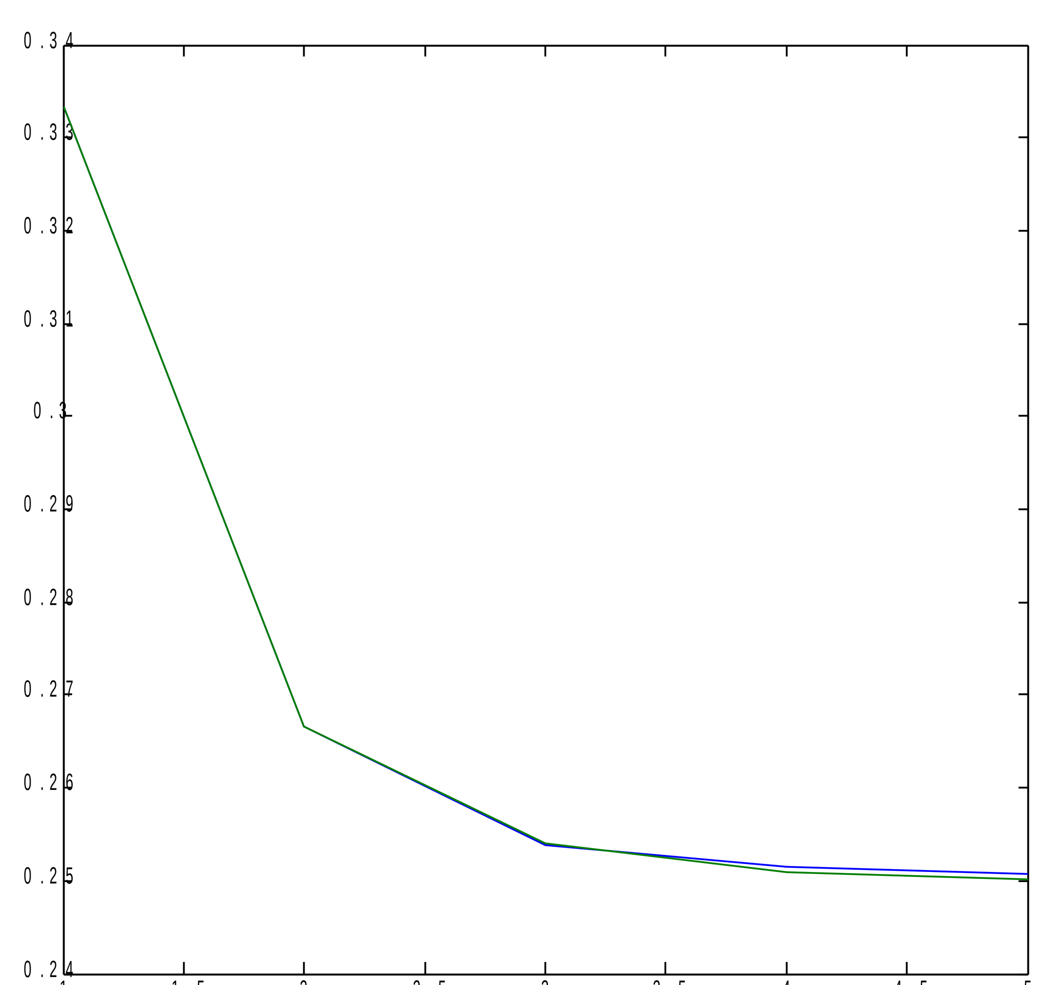


Nonergodic Island Map
y greater than .5, map is non-ergodic
y less than .5, map is ergodic



Graph of defect vs scale for computer generated answers and theoretical answers

Nonergodic Island Map
Defect vs Scale



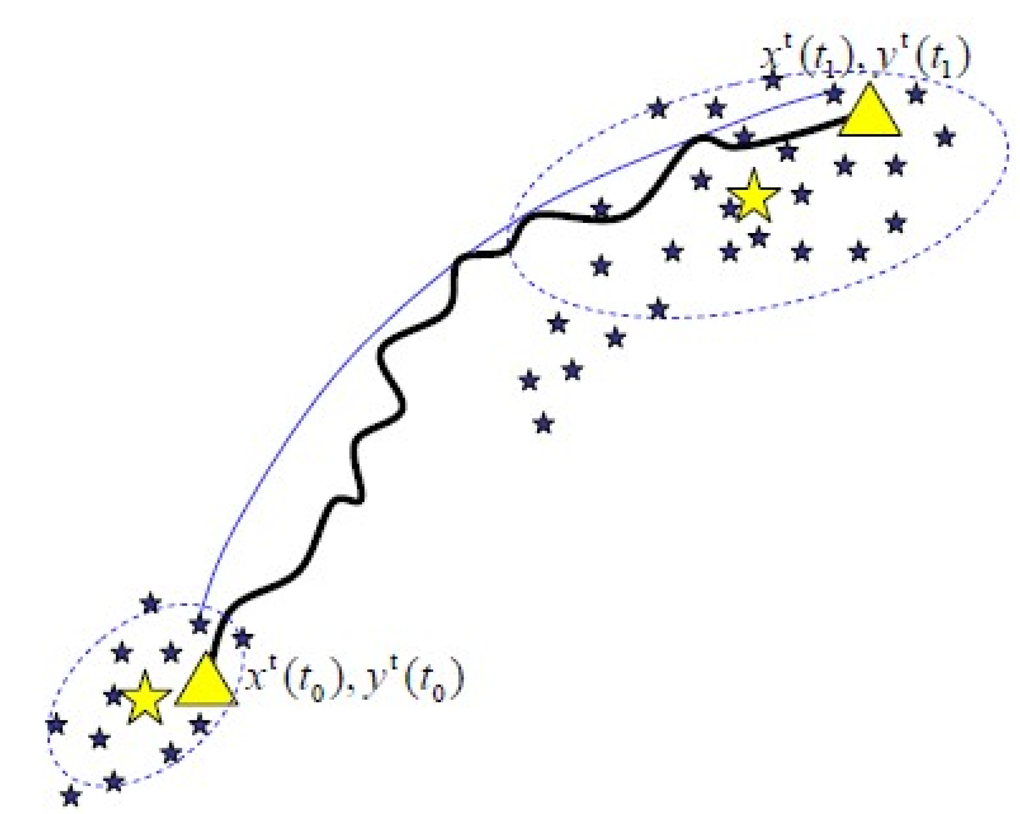
As shown above, the deviation from the computer generated numbers and the theory generated numbers is very slight

Particle Filters

What is a particle filter?
A way of combining model predictions and observation to get an estimate of a system's state

We are interested in systems where particle trajectories follow complicated nonlinear paths

How does it work?
1. Begin with distribution (particle cloud) of initial conditions
2. This distribution is a set of discrete state values which are weighted in order to reflect uncertainty in the initial conditions
3. Move particles forward in time
4. Reweight according to new positions relative to an observation i.e. particles that are closer to the observation get higher weights
5. Repeat process for subsequent observations



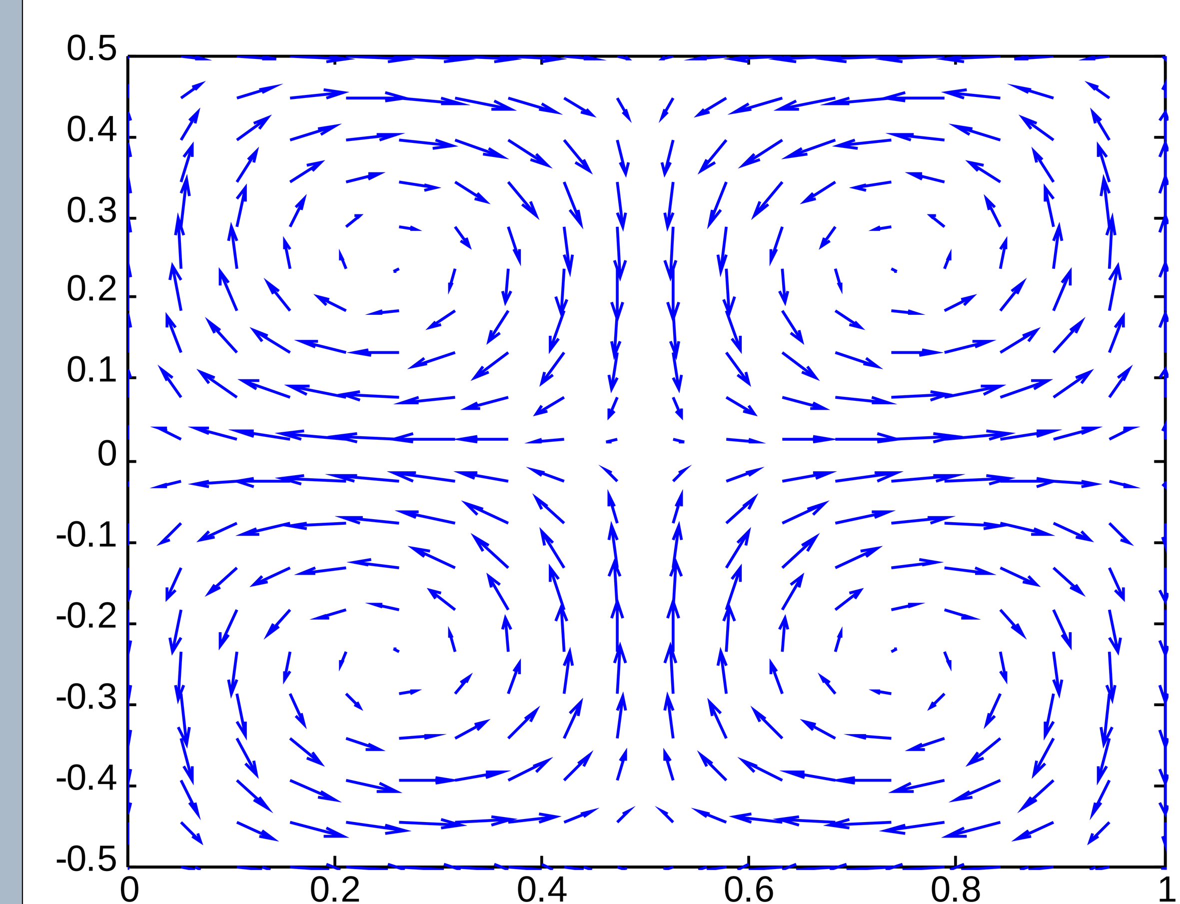
Use this method to estimate flow paths

·This method can break down if few or no particles in the cloud are near an observation value
·Models can be expensive to compute, so we would like to use as few particles as possible to accurately filter
-Monitoring the ergodic defect may indicate instances when a more robust filter is needed

Shallow Water Equations

What are shallow water equations?
A set of partial differential equations for u (horizontal velocity), v (vertical velocity), and h (height) to describe flow trajectories $\dot{x} = u$, and $\dot{y} = v$.

Produces flow fields such as the one below



The flow field changes with time and therefore so do the trajectories

Using a particle filter, we can determine the paths of trajectories in the flow fields, and track the movement of the system

Future Work

Combining filters and ergodicity work

·Help to determine when a filter fails
· Increase accuracy of filter
· better filter monitoring
· Development of a more sophisticated filter

References

Apte, A., Jones, C. K. R. T., Stuart, A. M. "A Bayesian approach to Lagrangian data assimilation." *Tellus*, 2008.

Scott, Sherry E., *et al*. "Capturing deviation from ergodicity at different scales." *Physica D*, 2009.